College and Career Mathematics
Winter Student Enrichment Packet

Detroit Public Schools Community District
Department of Curriculum and Instruction
Office of Mathematics
NOTE TO STUDENT

This Winter Student Enrichment Packet has been compiled to complement high school mathematics classroom instruction aligned to Maryland College and Career Ready Standards (MCCRS). It is intended to be used for review and practice of previously taught and new concepts.

We strongly encourage you to work diligently to complete the activities in this packet. You may experience some difficulty with some problems in this packet, but we encourage you to explore and attempt to find a solution to the best of your ability.
1) What is the sum of the complex numbers $2 + 3i$ and $4 + 8i$, where $i = \sqrt{-1}$?
   - A) 17
   - B) 17i
   - C) 6 + 11i
   - D) 8 + 24i

2) $4x^2 - 9 = (px + t)(px - t)$
   
   In the equation above, $p$ and $t$ are constants. Which of the following could be the value of $p$?
   - A) 2
   - B) 3
   - C) 4
   - D) 9

3) The vertex of the parabola in the $xy$-plane above is $(0, c)$. Which of the following is true about the parabola with the equation $y = -a(x-b)^2 + c$?
   - A) The vertex is $(b, c)$ and the graph opens upward.
   - B) The vertex is $(b, c)$ and the graph opens downward.
   - C) The vertex is $(-b, c)$ and the graph opens upward.
   - D) The vertex is $(-b, c)$ and the graph opens downward.

4) Which of the following is equivalent to $\frac{4x^2 + 6x}{4x + 2}$?
   - A) $x$
   - B) $x + 4$
   - C) $x - \frac{2}{4x + 2}$
   - D) $x + 1 - \frac{2}{4x + 2}$
5) The table above shows the flavors of ice cream and the toppings chosen by the people at a party. Each person chose one flavor of ice cream and one topping. Of the people who chose vanilla ice cream, what fraction chose hot fudge as a topping?

A) \( \frac{8}{25} \)
B) \( \frac{5}{13} \)
C) \( \frac{13}{25} \)
D) \( \frac{8}{13} \)

6) The scatterplot below shows the amount of electric energy generated, in millions of megawatt-hours, by nuclear sources over a 10-year period.

Of the following equations, which best models the data in the scatterplot?

A) \( y = 1.674x^2 + 19.76x - 745.73 \)
B) \( y = -1.674x^2 - 19.76x - 745.73 \)
C) \( y = 1.674x^2 + 19.76x + 745.73 \)
D) \( y = -1.674x^2 + 19.76x + 745.73 \)

7) 

\[
\begin{align*}
7x + 3y &= 8 \\
6x - 3y &= 5
\end{align*}
\]

For the solution \((x, y)\) to the system of equation above, what is the value of \(x - y\)?

A) \(-\frac{4}{3}\)
B) \(\frac{2}{3}\)
C) \(\frac{4}{3}\)
D) \(\frac{22}{3}\)

8) 

<table>
<thead>
<tr>
<th>(x)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>(\frac{11}{4})</td>
<td>(\frac{25}{4})</td>
<td>(\frac{39}{4})</td>
<td>(\frac{53}{4})</td>
<td>(\frac{67}{4})</td>
</tr>
</tbody>
</table>

Which of the following equations relates \(y\) to \(x\) for the values in the table above?

A) \( y = \frac{1}{2} \cdot \left(\frac{5}{2}\right)^x \)
B) \( y = 2 \cdot \left(\frac{3}{4}\right)^x \)
C) \( y = \frac{3}{4}x + 2 \)
D) \( y = \frac{7}{2}x - \frac{3}{4} \)
9) Triangles $ABC$ and $DEF$ are shown above. Which of the following is equal to the ratio $\frac{BC}{AB}$?

A) $\frac{DE}{DF}$
B) $\frac{DF}{DE}$
C) $\frac{DF}{EF}$
D) $\frac{EF}{DE}$

10) In 2008, there were 21 states with 10 or more electoral votes, as shown in the table. Based on the table, what was the median number of electoral votes for the 21 states?

A) 13
B) 15
C) 17
D) 20
Paul started to train for a marathon. The table shows the number of miles Paul ran during each of the first three weeks after he began training.

<table>
<thead>
<tr>
<th>Week</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance (miles)</td>
<td>10</td>
<td>12</td>
<td>14.4</td>
</tr>
</tbody>
</table>

If this pattern continues, which of the listed statements could model the number of miles Paul runs \( a_n \), in terms of the number of weeks, \( n \), after he began training?

Select all that apply.

A) \( a_n = 10 + 2(n - 1) \)
B) \( a_n = 10n^2 \)
C) \( a_n = 10(1.2)^{n-1} \)
D) \( a_1 = 10, a_n = 1.2a_{n-1} \)
E) \( a_1 = 10, a_n = 2 + a_{n-1} \)

Which graph best represents the solution to this system of inequalities?

\[
\begin{align*}
x + y & \leq 6 \\
x + 2y & \leq 8
\end{align*}
\]
13) The figure shows a graph of the function $f(x)$ in the $xy$-coordinate plane, with the vertex at (1, 9) and the zeroes at -2 and 4.

![Graph of $f(x)$]

The function $g$ is defined by $g(x) = -3x + 2$. Which statements are true? Select all that apply.

A) $f(-2)$ is greater than $g(-2)$.
B) $f(-1)$ is less than $g(-1)$.
C) $f(0)$ is greater than $g(0)$.
D) $f(1)$ is less than $g(1)$.
E) $f(2)$ is greater than $g(2)$.

14) The apothem of a regular polygon is the distance from the center to any side.

![Diagram of an apothem]

If the length of the apothem remains constant at 10 inches, the formula for the perimeter of a regular polygon as a function of the number of sides $n$ is $P(n) = 10 \left( \tan \frac{360^\circ}{2n} \right) (2n)$.

As the regular polygon changes from a pentagon (5 sides) to an octagon (8 sides), what is the approximate average rate of change in the perimeter?

A) decrease of 0.80 inches for each additional side
B) decrease of 2.13 inches for each additional side
C) decrease of 4.56 inches for each additional side
D) decrease of 6.38 inches for each additional side
The diagram shows two cylinders with bases that have the same center and heights of 12 millimeters.

**Part A**
Which is a function for the volume, \( V \), that is inside the larger cylinder but outside the one with the smaller radius, \( r \)?

A) \( V(r) = 1,200\pi - 12\pi r^2 \)
B) \( V(r) = 120\pi - 12\pi r^2 \)
C) \( V(r) = 12\pi r^2 \)
D) \( V(r) = 12\pi (10 - r)^2 \)

**Part B**
Suppose that there is space between the inner and outer cylinders and the radius of the inner cylinder must be an integer greater than or equal to 3. What is the domain of \( V \)?

A) all integers greater than or equal to 3
B) 3, 4, 5, 6, 7, 8, 9, or 10
C) 3, 4, 5, 6, 7, 8, or 9
D) \( 3 \leq r \leq 9 \)

The polynomial \( p(x) = 2x^3 + 13x^2 = 17x - 12 \) has \( (x + 4) \) as a factor. Factor the polynomial into three linear factors. Describe the steps you would use to sketch the graph of the function defined by this polynomial. Identify all intercepts and describe the end behavior of the graph.
Consider the polynomial function \( f(x) = (2x - 1)(x + 4)(x - 2) \).

**Part A**
What is the y-intercept of the graph of the function in the xy-coordinate plane?

**Part B**
For what values of \( x \) is \( f(x) > 0 \)?

Select all that apply.

A) \( x < -4 \)
B) \( -4 < x < \frac{1}{2} \)
C) \( -4 < x < 2 \)
D) \( \frac{1}{2} < x < 2 \)
E) \( x > \frac{1}{2} \)
F) \( x > 2 \)

**Part C**
What is the end behavior of the graph of the function?

A) As \( x \to -\infty, f(x) \to \infty \), and as \( x \to \infty, f(x) \to \infty \).
B) As \( x \to -\infty, f(x) \to \infty \), and as \( x \to \infty, f(x) \to -\infty \).
C) As \( x \to -\infty, f(x) \to -\infty \), and as \( x \to \infty, f(x) \to \infty \).
D) As \( x \to -\infty, f(x) \to -\infty \), and as \( x \to \infty, f(x) \to -\infty \).

**Part D**
How many relative maxima does the function have?

A) none
B) one
C) two
D) three
Part A
The histograms show the distribution of heart rates of randomly selected adult males between the ages of 40 and 45 after 20 minutes of continuous exercise. The adult males were randomly assigned to use either a new elliptical machine (Experimental Group) or a traditional treadmill machine (Control Group).

What conclusion about the difference between the distributions of the heart rates for these two groups can be drawn? Justify your answer.
Part B
After the participants worked out three times per week for four weeks solely on their assigned machines, participants’ heart rates were collected again after 20 minutes of continuous exercise. The data are shown in the histograms.

What conclusion about the difference between the distributions of the heart rates for the two groups can be drawn? Justify your answer.

If the target heart rate range for adult males aged between 40 and 45 after 20 minutes of exercise is around 175 beats per minute, what recommendation would you make in terms of which machine to use? Justify your answer.

Based upon these data, what conclusion about exercise machines in general can be made?
19) **Part A**
A bank offers a savings account that accrues simple interest annually based on an initial deposit of $500. If $S(t)$ represents the money in the account at the end of $t$ years and $S(5) = 575$, write a function that could be used to determine the amount of money in the account over time. Show your work or explain your reasoning.

**Part B**
Another bank offers a savings account that accrues compound interest annually at a rate of 3%. What is the initial amount needed in this account so that it will have the same amount of money at the end of 10 years as the account in Part A at the end of 10 years? Show your work or explain your reasoning.

20) **Phil and Matt made cookies for a fundraiser at their high school.**

- Phil made 25% more cookies than Matt.
- The cookies sold for $0.25 each.
- After the sale, 20% of the combined total of their cookies remained.

**Part A**
Create an equation to represent the total amount of money Matt and Phil earned at the fundraiser based on the number of cookies Matt made. Explain how you determined your equation.

**Part B**
Phil and Matt made a total of $72.00 selling the cookies. How many cookies did Phil make and how many cookies did Matt make? Show your work.

**Part C**
Next year, Phil and Matt may sell the cookies for $0.50 each. They plan to make the same total number of cookies, but they predict that they will only sell 70% of them given the price increase. Based on their prediction, should Phil and Matt raise the price of the cookies? Justify your answer.