## Mathematics

# Pre-Calculus

DETROIT
PUBLIC SCHOOLS
COMMUNITY DISTRICT

**Letter to Families from the DPSCD Office of Mathematics**

Dear DPSCD Families,

The Office of Mathematics is partnering with families to support Distance Learning while students are home. We empower you to utilize the resources provided to foster a deeper understanding of grade-level mathematics.

In this packet, you will find links to videos, links to online practice, and pencil-and-paper practice problems. The Table of Contents shows day-by-day lessons from April 14th to June 19th. We encourage you to take every advantage of the material in this packet.

Daily lesson guidance can be found in the table of contents below. Each day has been designed to provide you access to materials from Khan Academy and the academic packet. Each lesson has this structure:

| **Watch: Khan Academy (if internet access is available)** | **Practice: Khan Academy (if internet access is available)** | **Pencil & Paper Practice: Academic Packet** |
|---|---|---|
| Watch and take notes on the lesson video on Khan Academy | Complete the practice exercises on Khan Academy | Complete the pencil and paper practice. |

If one-on-one, live support is required, please feel free to call the **Homework Hotline** at 1-833-466-3978. Please check the Homework Hotline page for operating hours. We have DPSCD mathematics teachers standing by and are ready to assist.

We appreciate your continued dedication, support and partnership with Detroit Public Schools Community District and with your assistance we can press forward with our priority: Outstanding Achievement. Be safe. Be well!

Deputy Executive Director of K-12 Mathematics

# Important Links and Information

## Clever

Students access Clever by visiting www.clever.com/in/dpscd.

**What are my username and password for Clever?**
Students access Clever using their DPSCD login credentials. Usernames and passwords follow this structure:

Username: studentID@thedps.org
*Ex. If Aretha Franklin is a DPSCD student with a student ID of 018765 her username would be 018765@thedps.org.*

Password:
First letter of first name in upper case
First letter of last name in lower case
2-digit month of birth
2-digit year of birth
01 (male) or 02 (female)
*For example: If Aretha Franklin's birthday is March 25, 1998, her password and password would be Af039802.*

## Accessing Khan Academy

To access Khan Academy, visit www.clever.com/in/dpscd. Once logged into Clever, select the Khan Academy button:

Khan Academy

## Accessing Your CPM eBook

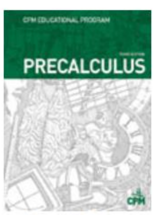
Students can access their CPM eBook in two ways:

**Option 1: Access the eBook through Clever**

1. Visit www.clever.com/in/dpscd. Login using your DPSCD credentials above.
2. Click on the CPM icon:

**Option 2: Visit http://open-ebooks.cpm.org/**

1. Visit the website listed above.
2. Click "I agree"
3. Select the CPM Precalculus eBook:




**Precalculus**
**Third Edition**

## Desmos Online Graphing Calculator

Access to a free online graphing and scientific calculator can be found at https://www.desmos.com/calculator.

## Table of Contents

In the following table, you will find the table of contents and schedule for the week of April 13, 2020 through the week of June 15, 2020.

| Week | Date | Topic | Watch (10 minutes) | Online Practice (10 minutes) | Pencil & Paper Practice (25 minutes |
|---|---|---|---|---|---|
| **Week 1** **04/13-04/17** **5 Days** | Day 1 | Holiday | N/A | N/A | N/A |
| | Day 2 | Lesson 3.1.1: Operations with Rational Expressions | Video: Adding Rational Exp. Video: Subtracting Rational Exp. | Practice: Adding and Subtracting Rational Expressions | Problems 1 - 8 |
| | Day 3 | Lesson 3.1.2: Rewriting Expressions and Equations | Video: Rewriting Expressions | Practice: Rewriting Expressions | Problems 1-6 |
| | Day 5 | Lesson 3.1.3: Solving Nonlinear Systems of Equations | Video: Nonlinear Systems | Practice: Nonlinear Systems | Problems 1-3 |

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| | | | Video: Nonlinear Systems | | |
| | Day 5 | Lesson 3.1.4: Polynomial Division | Video: Polynomial Division<br>Video: Polynomial Division | Practice: Polynomial Division<br>Practice: Polynomial Division Part 2 | Problems 1-5 |
| **Week 2**<br>**04/20-04/24**<br>**5 Days** | Day 1 | Lesson 3.1.5: Solving Classic Word Problems | Video: Average Rate | | Problems 1-5 |
| | Day 2 | Lesson 3.2.1: Using Sigma Notation | Video: Sigma Notation | Practice: Sigma Notation | Problems 1-5 |
| | Day 3 | Lesson 3.2.2: Area under a Curve (Part I) | Video: Approximating the Area Under a Curve<br>Article: left and Right Rectangle Approximation | Practice: Approximating the Area Under a Curve | Problems 1-4 |

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| | Day 4 | Lesson 3.2.3: Area under a Curve (Part II) | Video: Approximating the Area Under a Curve (Part II) | | Problems 1 - 4 |
| | Day 5 | Lesson 3.2.4: Area under a Curve (Part III) | Video: Approximating the Area Under the Curve (Part III) | Practice: Approximating Area Under the Curve (Part III) | Problems 9-11 |
| **Week 3**<br><br>**04/27-05/01**<br><br>**5 Days** | Day 1 | Lesson 3.2.Extra U-Substitution | Video: Factoring Using U-Substitution | Practice: Factoring Using U-Substitution | Problems 1-18 |
| | Day 2 | Lesson 4.1.1: Graphs of Polynomials in Factored Form | Video: Polynomial Graphs in Factored Form | Practice: Graphs of Polynomials in Factored Form | Problems 1-11 |
| | Day 3 | Lesson 4.1.2: Writing Equations of Polynomial Functions | Video: Roots of a Polynomial<br><br>Video: Matching Roots with the Function | Practice: Roots of a Polynomial | Problems 1-8 |

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| | Day 4 | Lesson 4.1.3: Identifying and Using Roots of Polynomials | Video: Identifying Roots | Practice: Identifying Roots | Problems 1-3, 14, 15 |
| | Day 5 | Lesson 4.2.1: Graphing Transformations of $y = \frac{1}{x}$ | Video: Graphing Transformations | Practice: Graphing Transformations | Problems 1-5 |
| **Week 4**<br><br>**05/04-05/08**<br><br>**5 Days** | Day 1 | Lesson 4.2.2: Graphing Rational Functions | Video: Graphing Rational Functions | Practice: Graphing Rational Functions | Problems 6-11 |
| | Day 2 | Lesson 4.2.3: Graphing Reciprocal Functions | Video: Polynomial End Behavior | Practice: Polynomial End Behavior | Problems 1-5 |
| | Day 3 | Lesson 4.3.1: Polynomial and Rational Inequalities | Video: Polynomial and Rational Inequalities | Practice: Polynomial and Rational Inequalities | Problems 1-7 |
| | Day 4 | Lesson 4.3.2: Applications of Polynomial and Rational Functions | Video: Horizontal Asymptotes<br><br>Video: Vertical Asymptotes | Practice: Horizontal and Vertical Asymptotes | Problems 1-5 |

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| | Day 5 | Chapter 4 Closure | N/A | Practice: Rational Functions Quiz | N/A |
| **Week 5**<br><br>**05/11-05/15**<br><br>**5 Days** | Day 1 | Lesson 5.1.1: Applications of Exponential Functions | Video: Writing Exponential Equations from two points<br><br>Video: Compound Interest | Practice: Writing Exponential Functions from two points | Problems 1-7 |
| | Day 2 | Lesson 5.1.2: Stretching Exponential Functions | Video: Graphing Horizontal Shifts | | Problems 1-5 |
| | Day 3 | Lesson 5.1.3: The number e | Video: Logarithmic functions to Exponential | Practice: Logarithmic functions to Exponential | Problems1 -5 |
| | Day 4 | Lesson 5.2.1: Logarithms | Video: Writing Inverse functions | Practice: Graphing logarithmic functions | Problems 1-8 |
| | Day 5 | Lesson 5.2.2: Properties of Logarithms | Video: Introduction to Logarithms part 1 | Practice: Properties of Logarithms | Problems 1-12 |

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| | | | Video: Introduction to Logarithms part 2<br><br>Summary: Properties of Logs | | |
| **Week 6**<br><br>**05/18-05/22**<br><br>**5 Days** | Day 1 | Lesson 5.2.3: Solving Exponential and Logarithmic Equations | Video: Logarithmic Product Rule<br><br>Video: Logarithmic power Rule<br><br>Video: Using properties of Logarithms | Practice: Use the properties of Logarithms | Problems 1-15 |
| | Day 2 | Lesson 5.2.4: Graphing Logarithmic Functions | Video: Solving Exponential Equations | Practice: Solving Exponential equations using Logarithms | Problems 1-14 |
| | Day 3 | Lesson 5.2.5: Applications of Exponentials and Logarithms | Video: Solving Logarithmic Equations | Practice: Solving logarithmic equations | Problems 1-11 |

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| | Day 4 | Chapter 5 Closure<br><br>Lesson 7.1.1: An Introduction to Limits | Closure<br><br>Introduction to Limits | Unit TEST (try your best)<br><br>Practice with Limits | |
| | Day 5 | Lesson 7.1.2: Working with One Sided Limits | One Sided Limits | One Sided Limits | Problems 1-2 |
| **Week 7**<br><br>**05/25-05/29**<br><br>**5 Days** | Day 1 | Holiday | N/A | N/A | N/A |
| | Day 2 | Lesson 7.1.3: The Definition of a Limit | Connecting limits and graphs<br><br>Unbounded Limits | Practice with limits and end behavior | Problems 1-15 |
| | Day 3 | Lesson 7.1.4: Limits and Continuity | Connecting limits and graphs (2) | Quiz on Limits | Problems 1-6 |
| | Day 4 | Lesson 7.1.5: Special Limits | Limits of Trig Functions | Practice with Trig Limits | |

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| | Day 5 | Lesson 7.2.1: Rates of Change from Data | Introduction to Average Rate of Change<br>Approximating Limits using Tables<br>Estimating Limits from Table<br>One sided limits from tables | Practice Creating tables for approx. limits<br>Practice Estimating Limits from Tables<br>Practice with one sided limits from tables | Problems 1-4 |
| **Week 8**<br><br>**06/01-06/05**<br><br>**5 Days** | Day 1 | Lesson 7.2.2: Slope and Rates of Change | Average Rate of Change from a Graph<br>Worked Ex: Avg Rate of Change from a Table | Practice: Average Rate of Change Tables and Graphs | Problems 1,2,7,3,4 |
| | Day 2 | Lesson 7.2.3: Average Velocity and Rates of Change | Rates of Change (U. Bolt) | | Problems 1-4 |
| | Day 3 | Lesson 7.2.4: Moving from AROC to IROC | Derivative as a Concept | Practice Derivative as a Limit | Problems 1-4 |

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| | | | Formal Definition of Derivative | | |
| | Day 4 | Lesson 7.2.5: Rate of Change Applications | Interpreting Derivatives in Context | Practice Interpreting Derivatives in Context | Problems 1-4 |
| | Day 5 | Chapter 7 Closure | Strategy in Finding Limits | Practice in Finding Limits | |
| **Week 9**<br>**06/08-06/12**<br>**5 Days** | Day 1 | Lesson 8.1.1: Graphing Transformations of the Sine Function | Graph of sin x | Amplitude of Sine Functions | Problems 1-5 |
| | Day 2 | Lesson 8.1.2: Modeling with Periodic Functions | Amp and Period of Sine Functions<br>Modeling with Trig | Practice with Amp and Period Problems<br>Practice with Modeling and Trig | Problems 1-5 |
| | Day 3 | Lesson 8.1.3: Improving the Spring Problem | Transform Sin Graphs (vertical/reflection) | Graph Sin Functions | Problems 1-4 |

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| | | | Transform Sin Graph (horizontal stretch) | Phase Shifts<br>Phase Shifts | |
| | Day 4 | Lesson 8.2.1: Graphing Reciprocal Trigonometric Functions | Finding Reciprocal Trig Functions | Reciprocal Trig Ratios | Problems 1-5 |
| | Day 5 | Lesson 8.3.1: Simplifying Trigonometric Expressions | Using Trig Identities<br>Trig Ratio Reference Sheet | | Problems 1-6 |
| **Week 10**<br>**06/15-06/19**<br>**5 Days** | Day 1 | Lesson 8.3.2: Proving Trigonometric Identities | Trig Identity Example Proof | Practice: Trig Identities Challenge | Problems 1-10 |
| | Day 2 | Lesson 8.3.3: Angle Sum and Difference Identities | Review of Trig Angle Sum Identities | Using Trig Angle Addition Identities | Problems 1-5, 1-3 |

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| | Day 3 | Lesson 8.3.4: Double-Angle and Half-Angle Identities | Using Double Angle Cos Identity | | Problems 2-5 |
| | Day 4 | Lesson 8.3.5: Solving Complex Trigonometric Equations | Using Trig Angle Identities | Trig Practice Problems Quiz | Problems 1-7 |
| | Day 5 | Chapter 8 Closure | Trigonometric Identities Practice Test | Trigonometric Identities Practice Test | |

**Week 1, Day 1**
**Lesson 3.1.1 Operations with Rational Expressions**
**Students will add, subtract, multiply, and divide rational expressions.**

**1.** Simplify the following rational expressions.

a. $\dfrac{x-1}{5}+\dfrac{3x+7}{x+2}$

b. $\dfrac{x^3+2x^2}{x+1}\div\dfrac{x^2+2x}{x^2-1}$

**2.** Simplify the following rational expressions.

a. $\dfrac{5x-2}{2x+3}-\dfrac{8}{3}$

b. $\dfrac{a-3}{a^2+2a-8}\cdot\dfrac{a^{2+}4a}{3a^2-27}$

**3.** Simplify the following rational expressions.

a. $\dfrac{x^2-1}{2x^2-11x+5}\div\dfrac{3x^2+9x+6}{2x^2+3x-2}$

b. $\dfrac{6}{x-3}+\dfrac{8}{x+4}$

**4.** Simplify. $\dfrac{10}{x-y}-\dfrac{6y}{x^2-y^2}$

**5.** Simplify. $\dfrac{1}{x+2}+\dfrac{1}{x^2-4}-\dfrac{2}{x^2-x-2}$

**6.** Simplify. $\dfrac{x^2+x-2}{x^2+2x-3}+\dfrac{x-1}{x+3}$

**7.** Simplify. $\dfrac{1}{x+3}-\dfrac{1}{x^2-9}+\dfrac{2}{x^2+4x+3}$

**8.** Simplify. $\dfrac{1}{x+8}-\dfrac{1}{x^2+4x-32}+\dfrac{5}{x^2-11x+28}$

**Week 1, Day 2**
**Lesson 3.1.2: Rewriting Expressions and Equations**
**Students will simplify complex fractions and use substitution to simplify and factor algebraic expressions.**

**1.** Simplify the following complex fraction. $\dfrac{\frac{1}{x^2}+y^2}{x^2-\frac{1}{y^2}}$

**2.** Simplify the following complex fraction. $\dfrac{\frac{x}{y^2}+\frac{1}{x}}{\frac{y}{x^2}+y^2}$

**3.** Simplify the following complex fraction. $\dfrac{x+y}{\frac{1}{x}+\frac{1}{y}}$

**4.** Simplify. $\dfrac{x^{-4}y^3+x^2y^{-3}}{x^{-1}-y^{-1}}$

**5.** Simplify. $\dfrac{4x^{-2}+x^{-3}}{2x^{-3}-x^{-4}}$

**6.** Simplify. $\dfrac{\frac{1}{x^2}-\frac{1}{y^2}}{\frac{-3}{x^2}+\frac{1}{xy}+\frac{2}{y^2}}$

**Week 1, Day 3**
**Lesson 3.1.3 Solving Nonlinear Systems of Equations**
**Students will solve nonlinear systems of equations.**

**1.** Solve the system: $\begin{cases} x^2+y^2=10 \\ \dfrac{1}{x}-\dfrac{3}{y}=0 \end{cases}$

**2.** Solve the system: $\begin{cases} 4x^2+5y^2=112 \\ xy=12 \end{cases}$

**3.** Solve the system: $\begin{cases} |x|+y=12 \\ x^2-3y^2=54 \end{cases}$

**4.** Solve the system: $\begin{cases} \sqrt{-x}-4y=-1 \\ x-4y=-7 \end{cases}$

**Week 1, Day 4**
**Lesson 3.1.4 Polynomial Division**
**Student will Divide Polynomials.**

**1.** Divide: $\dfrac{x^3-4x^2+x+6}{x-2}$

**2.** Divide: $\dfrac{6x^5-x^4-5}{x-1}$

**3.** Divide $P(x) = x^4 + x^2 - 5x + 1$ by $x - 1$.

**4.** Is $x - 5$ a factor of $x^3 - 3x^2 - 6x - 20$? Explain your reasoning.

**5.** Divide: $\dfrac{3x^4-2x^2-1}{x+2}$

**Week 2, Day 1**
**Lesson 3.1.5 Solving Classic Word Problems**
**Students will use a variety of strategies to solve classic types of word problems.**

**1.** Sally rides her bike to run errands. It takes her 20 minutes at 15 mph to get to her first stop. She spends 15 minutes riding at 12 mph to get to her second stop. It then takes her 18 minutes riding at 10 mph to get home.

a. Draw a speed vs. time graph for the time that Sally spent riding her bike.

b. How far did she travel?

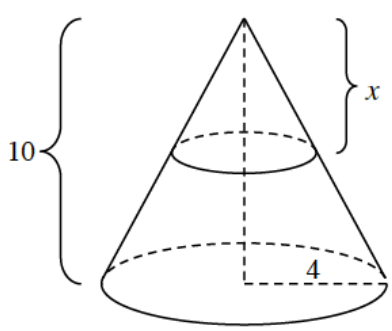
c. What was her average speed?

**2.** The lengths of the sides of a right triangle are given by $x$, $x+4$, and $x+8$. What are the lengths of the sides?

**3.** A stick 4 feet tall casts a shadow 5 feet long at 6:36 p.m. How tall is a pole that casts a shadow 32 feet long at the same time?

**4.** Fleta wants to earn $2000 in interest on her investments this year. She currently has $35,000 in an account that earns 3% annual interest, but she is not allowed to add to this account. She has more money to invest and the best rate the bank can give her is 2.25% annual interest. How much does she need to invest in the new account to meet her desired earnings?

**5.** A cone has a height of 10 inches and a radius of 4 inches. If a plane cuts the cone $x$ inches below the cone's peak, what is the area of the circular cross section?



**Week 2, Day 2**
**Lesson 3.2.1 Using Sigma Notation**
**Students will recognize and be able to calculate sums by expanding sigma notation as well as write finite arithmetic series in sigma notation.**

**1.** Expand. $\sum_{n=2}^{5}\left(4n^3-1\right)$

**2.** Expand. $\sum_{k=1}^{4}\left(3k^2+5\right)$

**3.** Write the following sum in expanded form. $\sum_{n=3}^{7}(4n-7)$

**4.** Write the given expression using sigma notation.

$$0.4\left(\frac{1}{2}+\frac{1}{2.4}+\frac{1}{2.8}+\frac{1}{3.2}+\frac{1}{3.6}\right)$$

**5.** Write the given expression using sigma notation.

$0.2(4^{3} + 4^{3.2} + 4^{3.4} + \ldots + 4^{4.8})$

**Week 2, Day 3**
**Lesson 3.2.2 Area under a Curve (Part I)**
**Students will estimate the area under a curve and understand that area under a velocity curve represents distance.**

**1.** Let $g(x) = (x - 2)^2$. Approximate $A(g,\ 2 \le x \le 4)$ using right endpoint rectangles of width 0.5 units. Express your sum using sigma notation.

**2.** Given $f(x) = \frac{4}{x+1}$, approximate $A(f(x),\ 0 \le x \le 3)$ using 6 right endpoint rectangles.

**3.** Given $f(x)= 2x^2$, approximate $A(f,\ 1 \le x \le 4)$ using 5 left endpoint rectangles.

**4.** Given $g(x) = \sqrt{x-2}$ , approximate $A(g,\ 5 \le x \le 7)$ using ten left endpoint rectangles.

**Week 2, Day 4**
**Lesson 3.2.3 Area under a Curve (Part II)**
**Students will approximate area under a curve using left endpoint and right endpoint rectangles. They will use sigma notation to express their approximations.**

**1.** Write the sigma notation for approximating the area under the curve $f(x) = x^2 + x - 12$ for $4 \le x \le 10$ using 20 right endpoint rectangles.

**2.** Given $g(x) = \frac{x+1}{x-2}$, approximate $A(g, 2 \le x \le 5)$ using 8 right endpoint rectangles.

**3.** Approximate the area under the curve $y = -2x^2 + x + 6$ for $-1 \le x \le 2$ using 6 left endpoint rectangles.

**4.** Approximate the area under the curve $y = \frac{3x}{2x-1}$ for $-3 \le x \le 0$ using left endpoint rectangles of width 0.1.

**Week 2, Day 5**
**Lesson 3.2.4 Area under a Curve (Part III)**
**Students will practice approximating areas with left endpoint and right endpoint rectangles.**

**1.** Given $j(x) = \frac{1}{3}(x-1)^2 + 4$, approximate $A(j, 1 \le x \le 4)$ using 5 right endpoint rectangles. Sketch the curve, showing the rectangles used to determine the area. Use sigma notation to represent your approximation. Is the approximation an underestimate or an overestimate of the actual area? Explain.

**2.** Given $f(x) = 3x^2 - 6x$, approximate $A(f, 1 \le x \le 4)$ using 7 left endpoint rectangles.

a. Use sigma notation to show the sum.

b. Determine if the approximation is an underestimate or an overestimate of the actual area. Justify your answer using a sketch.

c. How can you change your sigma notation to estimate the area using right endpoint rectangles?

**3.** Sketch a graph of $h(x) = \frac{1}{x+5} + 7$ over the interval $-2 \le x \le 4$. Determine an underestimate of the area under the curve, for the given interval, using rectangles of width

**Week 3, Day 1**
**3.2.Extra U-Substitution**
**Students will factor expressions using substitution.**

**1.** Factor completely. $(x+y)^2-4(x+y)+4$

**2.** Factor completely. $4(a-b)^2-8(a-b)+4$

**3.** Factor completely. $m(n+q)^2-2m^2(n+q)$

**4.** Factor completely. $x^6-7x^3-8$

**5.** Factor completely. $x^{10}+x^5-12$

**6.** Factor completely. $x^8+2x^4-48$

**7.** Solve. $(x-2)^2-3(x-2)-10=0$

**8.** Solve. $x^{-2}-3x^{-1}-18=0$ **9.** Solve. $x^4-5x^2+4=0$

**Week 3, Day 2**
**Lesson 4.1.1 Graphs of Polynomials in Factored Form**
**Students will graph polynomial functions from equations given in factored form.**

**1.** Sketch the graph of $f(x) = (x+1)(x-2)(x+3)^2$ without using a graphing calculator. Do not scale your axes, but be sure to label the important points.

**2.** Sketch the graph of $f(x) = (x-3)(x+2)^2$ without using a graphing calculator. Do not scale your axes, but be sure to label the important points.

**3.** Sketch the graph of $f(x) = -(x+3)^2(x-4)$ without using a graphing calculator. Do not scale your axes, but be sure to label the important points.

**4.** Sketch the graph of $f(x) = -(x+1)(x+2)(x-3)$ without using a graphing calculator. Do not scale your axes, but be sure to label the important points.

**5.** Sketch the graph of $f(x) = -\frac{1}{9}(x-2)^2(x+3)^2$ without using a graphing calculator. Do not scale your axes, but be sure to label the important points.

**6.** The Mariana Trench, in the Pacific Ocean, is the deepest place on earth. If the x-axis represents the ocean basin (about 4.4 miles below sea level) the portion below the ocean basin of the Mariana trench can be modeled by $f(x) = 0.025(x-1)^3(x-7)$ where $f(x)$ is the depth below the ocean basin and x is horizontal distance, both in miles. Sketch the graph and describe it completely.

**Week 3, Day 3**
**Lesson 4.1.2 Writing Equations of Polynomial Functions**
**Students will write equations for the graphs of polynomial functions given the x-intercepts and one additional point.**

**1.** Write an equation for a $3^{rd}$ degree polynomial function in factored form, with real coefficients, that has roots at x = 3 $\pm\sqrt{5}$ and x = −1 and passes through the point (3, −10).

**2.** A $3^{rd}$ degree polynomial function has roots at x = –2*i* and x = 5. The *y*-intercept is (0, 25). Write an equation for this function in factored form with real coefficients.

**3.** A polynomial function of degree 3 contains the point (−1, 45), has an xintercept- of (4, 0), and has a root of x = −1 + $\sqrt{3}$. Write an equation for this function in factored form with real coefficients.

**4.** Write an equation for a $5^{th}$ degree polynomial function in factored form, with real coefficients, that has a double root at x = 3 and roots at x = 7 + 2*i* and x = −5. Write a possible equation for this function in factored form with real coefficients.

**5.** A $5^{th}$ degree polynomial function has roots at x = 3 − 2*i*, x = 1 + $\sqrt{5}$, and x = 8. The *y* intercept- is (0, 26). Write an equation for this function in factored form with real coefficients.

**Week 3, Day 4**
**Lesson 4.1.3 Identifying and Using Roots of Polynomials**
**Students will identify roots of polynomial functions**

**1.** Determine the roots of the given polynomial. Give *exact* answers. $p(x) = 3x^3 + 2x^2 - 8x$

**2.** Determine the roots of the given polynomial. Give *exact* answers. $p(x) = 3x^2 - 2x + 7$

**3.** Determine the roots of the given polynomial. Give *exact* answers. $p(x) = 2x^3 + 2x^2 + 13x$

**4.** Determine the roots of the given polynomial. Give *exact* answers. $p(x) = x^3 - 3x^2 - 50$

**5.** Let $f(x) = 2x^3 + 3x^2 - 7x - 4$.

a. Is x = 2 a root of the polynomial? Explain.

b. If x ≈ −1.7 is a local maximum and x ≈ 0.7 is a local minimum, what does that tell you about the graph of the function?

c. Determine all of the zeros of the function. Give exact answers.

**Week 3, Day 5**
**Lesson 4.2.1 Graphing Transformations of y =1/x**
**Students will rewrite rational expressions to transform functions in the form g(x) = (ax+b)/(x-c) into transformations of y = 1/(x-h) + k.**

**1.** Rewrite $f(x) = \frac{2x+5}{x+2}$ as a transformation of $g(x) = \frac{1}{x}$ and sketch the graph of $y = f(x)$.

**2.** Rewrite $f(x) = \frac{3x-7}{x-3}$ as a transformation of $g(x) = \frac{1}{x}$ and sketch the graph of $y = f(x)$.

**3.** Write a possible equation of a rational function that has a horizontal asymptote at *y* = 13 and a vertical asymptote at x = −2.

4. Write a possible equation of a rational function that has a horizontal asymptote at *y* = −12 and a vertical asymptote at x = 9.

5. Rewrite $f(x) = \frac{3x+8}{x+3}$ as a transformation of $g(x) = \frac{1}{x}$ and sketch the graph of $y = f(x)$.

**Week 4, Day 1**
**Lesson 4.2.2 Graphing Rational Functions**
**Students will graph rational functions with point discontinuities and slant asymptotes.**

1. Rewrite $f(x) = \dfrac{2x+5}{x-3}$ as a transformation of $g(x) = \dfrac{1}{x}$ and sketch the graph of $y = f(x)$.

**2.** Let $f(x) = \begin{cases} 3x^2 - 2x & \text{for } x < -\dfrac{\pi}{2} \\ \cos(x) & \text{for } -\dfrac{\pi}{2} \le x \le \pi \\ \dfrac{x^2 - 4x - 12}{x^2 - (6+\pi)x + 6\pi} & \text{for } x \ge \pi \end{cases}$

a. Sketch a graph of $y = f(x)$.

b. State the domain and range of $f$.

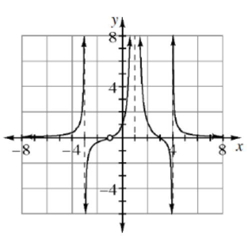
**3.** Consider the rational function $r(x) = \dfrac{x-4}{x+3}$ .

a. Determine the value of $n$, such that $f(x) = r(x) + n$ has a root at $x = -10$.

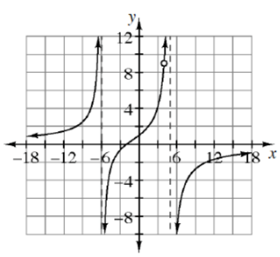
b. Rewrite $f(x)$ from part (a) as single-term expression.

**4.** Consider the rational function $r(x) = \dfrac{x-a}{x-b}$ . Explain what values of $a$ and $b$, if any would result in the function having no $x$- or $y$-intercepts. Justify your answer completely.

**5.** Write a possible equation for the rational function graphed below.




**6.** Write a possible equation for the rational function graphed below.



**Week 4, Day 2**
**Lesson 4.2.3 Graphing Reciprocal Functions**
**Students will graph y = 1/f(x) given an equation for the graph of y = f(x).**

**1.** Write the end-behavior function of $f(x) = \frac{23}{2x-1} + 54$.

**2.** Write the end-behavior function of $f(x) = \frac{35-2x}{2x+7}$.

**3.** Write the end-behavior function of $f(x) = \frac{61}{3x+19}$.

**4.** Write the end-behavior function of $f(x) = \frac{6x}{3x^2+1}$.

**5.** Write the end-behavior function of $f(x) = \frac{3x^2-8x+17}{2x^2-1}$.

**Week 4, Day 3**
**Lesson 4.3.1 Polynomial and Rational Inequalities**
**Students will solve polynomial and rational inequalities.**

**1.** Solve the inequality $x^2+4x>12$.

**2.** Solve: $6x^2 \le -11x-3$

**3.** Solve: $\frac{3(x-5)(x+1)}{x-7} < 0$

**4.** Solve the inequality $\frac{9(x-1)}{-7(x+6)(x+10)} \ge 0$.

**5.** Solve the inequality $10x^3+480x \ge 140x^2$.

**6.** Solve: $2x^3-40x^2 < 500-250x$

**7.** Solve: $\frac{x+7}{x+3} \le 5$

**Week 4, Day 4**
**Lesson 4.3.2 Applications of Polynomial and Rational Functions**
**Students will apply their knowledge of polynomial and rational functions to analyze everyday situations.**

**1.** State the locations of the asymptotes of $g(x)=\dfrac{2x-3}{x+7}$.

**2.** State the locations of the asymptotes of $h(x)=\dfrac{-5x+2}{x-5}$.

**3.** Identify all of the asymptotes of $y=\dfrac{x^3+5x^2+6x-3}{x+2}$.

**4.** State the locations of the asymptotes of $y=\dfrac{5x-1}{3x+2}$.

**5.** A baby bird is learning to fly. Its height above/below a branch can be modeled by the function $f(t) = 0.005t(t-2)^2(t-5)(t-8)$ for $0 \le t \le 10$, where $t$ is time in seconds and $f(t)$ is height in feet.

a. For what times is the bird on the branch?

b. When is the bird above the branch?

c. Where is the bird when $t = 7$?

**Week 4, Day 5**
**Lesson 5.1.1 Applications of Exponential Functions**
**Students will use exponential functions to model everyday situations.**

**1.** Write the equation of the exponential function with a horizontal asymptote of $y = 0$ that passes through the points (2, 36) and (4, 81).

**2.** Write the equation of the exponential function with a horizontal asymptote of $y = 0$ that passes through the points (1, 2) and (3, 18).

**3.** Write the equation of the exponential function with a horizontal asymptote of $y = 0$ that passes through the points (1, 36) and (3, 64).

**4.** A certain drug is removed from the human body according to an exponential model. Create a model for this drug given that 5 hours after taking the drug, there are 24 mg remaining in the body. After 2 more hours there are 18 mg left.

a. How much of the drug was present in the body after one hour?

b. When will there be 1 mg of the drug present in the body?

**5.** Recall the compound interest formula, $A = P\left(1+\frac{r}{n}\right)^{nt}$. How long will it take an investment earning 4.3% interest, compounded quarterly, to double?

**6.** Recall the compound interest formula, $A = P\left(1+\frac{r}{n}\right)^{nt}$. Miguel invested $2000 dollar in a savings account that is compounded monthly, one year ago. He now has $2025. What was the interest rate?

**7.** Recall the compound interest formula, $A = P\left(1+\frac{r}{n}\right)^{nt}$. Eva has invested $5000 in an account earning 5.4% interest, compounded daily. She will withdraw the money when she has made $2000 in interest. How long will she have to wait?

**Week 5, Day 1**
**Lesson 5.1.2 Stretching Exponential Functions**
**Students will understand that for exponential functions, a horizontal shift can be equivalently written as a vertical stretch.**

**1.** Rewrite the given equation in $y = a \cdot b^x$ form.

$y = 4(9)^{\frac{1}{2}x - 2}$

**2.** Rewrite the given equation in $y = a \cdot b^x$ form.

$y = 4(3)^{2x + 2}$

**3.** Rewrite the given equation in $y = a \cdot b^x$ form.

$y = 5\left(\frac{1}{2}\right)^{-3x + 2}$

**4.** Rewrite $5(3)^{2x - 4}$ in a $a \cdot b^x$ form.

**5.** Write given equation in $y = a \cdot b^x$ form.

$y = 20(2)^{3x - 2}$

**Week 5, Day 2**
**Lesson 5.1.3 The number e**
**Students will learn about e and will solve problems involving continuous growth.**

**1.** Rewrite each equation in the other (logarithmic/exponential) form.

a. $\log_2\left(\frac{1}{8}\right) = -3$

b. $A^y = B$

**2.** Rewrite each equation in the other (logarithmic/exponential) form.

a. $5^{-2} = \frac{1}{25}$

b. $\log_B(M) = K$

**3.** Rewrite each equation in the other (logarithmic/exponential) form.

a. $C = \log_V(I)$

b. $7^x = M$

**4.** Rewrite each equation in the other (logarithmic/exponential) form.

a. $d - 2 = \log_f(g)$

b. $4^x = n + 1$

**5.** Determine the inverse for the function below. Make sure that you express your answer using inverse function notation.

$f(x) = \log_3(x + 2)$

**Week 5, Day 3**
**Lesson 5.2.1 Logarithms**
**Students will practice converting between exponential and logarithmic equations.**

**1.** Determine the inverse equation of $f(x) = 4(5)^x - 3$.

**2.** Write a formula for the inverse of $f(x) = 3(4)^x + 5$.

**3.** Write a formula for the inverse of $f(x) = 6\log_7(x) - 8$.

**4.** Write a formula for the inverse of $y = -2\log_3(x) + 1$, then graph the original function.

**5.** The graph at right shows $y = f(x)$. Let $g(x) = 8^{x+1} + 5$.

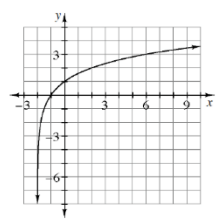
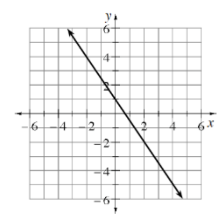
a. Evaluate $f^{-1}(g^{-1}(32))$.

b. Write an equation for $f(x)$.

**6.** The equation $y = \ln(f(x))$ is graphed at right.

a. What type of function is $f$?

b. Write an equation for $f(x)$.

**7.** Write the inverse of the function $f(x) = \log_{\sqrt{2}}(x)$. Demonstrate that $f$ and its inverse function are inverses using three different methods.

**8.** Keira is uncertain what a logarithm is and when to use logarithms to solve an equation. Provide a clear explanation for Keira. Include examples to illustrate your explanation.

**Week 5, Day 4**
**Lesson 5.2.2 Properties of Logarithms**
**Students will evaluate logarithms.**

**1.** Without a calculator, evaluate the following logarithmic expressions.

a. $\log_2(16)$ b. $\log_3\left(\frac{1}{9}\right)$ c. $\ln\left(e^5\right)$

**2.** Without a calculator, evaluate the following logarithmic expressions.

a. $\log_4(64)$ b. $\log_6\left(\frac{1}{36}\right)$ c. $\ln\left(\frac{1}{e}\right)$

**3.** Without a calculator, evaluate the following logarithmic expressions.

a. $\log_3(243)$ b. $\log_5(\sqrt{5})$ c. $\ln(1)$

**4.** Without a calculator, evaluate the following logarithmic expressions.

a. $\log_5(625)$ b. $\log_2(2\sqrt[4]{2})$ c. $3\ln\left(e^7\right)$

**5.** Without a calculator, evaluate the following logarithmic expressions.

a. $\log_8\left(\frac{1}{4}\right)$ b. $\ln\left(\frac{e^{3x+1}}{e^{-2}}\right)$

**6.** Without a calculator, evaluate the following logarithmic expressions.

a. $\ln\left(\sqrt{\frac{1}{e}}\right)$ b. $\log_{\sqrt{7}}(49)$

**7.** Simplify each logarithmic expression.

a. $\log_5(\log_2(32))$ b. $e^{\ln\sqrt{7}}$

**8.** Simplify each logarithmic expression.

a. $2^{\log_2(16)}$ b. $-2\ln\left(e^{3x}\right)$

**9.** Simplify each logarithmic expression.

a. $4\log_3(27^x)$ b. $5e^{3\ln(4)}$

**10.** Simplify each logarithmic expression.

a. $3\log_4(4^x)$ b. $\log(\log(10^{100}))$

**11.** Simplify each logarithmic expression.

a. $4^{\log_4(71)}$ b. $e^{2\ln(x-1)}$

**12.** Simplify each logarithmic expression.

a. $\ln\left(\frac{1}{e^x}\right)$ b. $3\log_4(4^{-5x})$

**Week 6, Day 1**
**Lesson 5.2.3 Solving Exponential and Logarithmic Equations**
**Students will solve equations with the variable in the exponent and equations with logarithms.**

**1.** Rewrite each of the expression using a single logarithm. $2\log(M)-3\log(N)$

**2.** Use the properties of logarithms to expand the following expression. $\log_a\left(\frac{x^2}{yz^7}\right)$

**3.** Use the properties of logarithms to expand the following expression. $\log_m\left(ab^2\sqrt{c^3}\right)$

**4.** Use the properties of logarithms to expand the following expression. $\log_t\left(h\sqrt{h^2+g}\right)$

**5.** Given $\log_b(K)=1.6$, $\log_b(J)=-0.4$, and $\log_b(L)=2.4$ determine the exact value of $\log_b\left(\frac{\sqrt{KL}}{J^3}\right)$.

**6.** Given $\log_b(K)=-0.6$, $\log_b(J)=-5.2$, and $\log_b(L)=7.3$ determine the exact value of $\log_b\left(\frac{KL^2}{Jb^3}\right)$.

**7.** Given $\log_b(K)=-3.6$, $\log_b(J)=2.4$, and $\log_b(L)=8.6$ determine the exact value of $\log_b\left(\frac{\sqrt[3]{(JK)^2}}{Lb^{-4}}\right)$.

**8.** Given $\log(A)=3.5$, $\log(B)=-1.6$, $\log(C)=0.4$, evaluate $\log\left(\frac{A\sqrt{C}}{B}\right)^3$.

**9.** Simplify: $3\log_7(2c)+\log_7(3d)-\frac{1}{2}\log_7(36)$

**10.** Simplify. $\log_7\left(x^2-2x-48\right)-\log_7(x-8)+\log_7(3)$

**11.** Simplify. $2\log_3(x-2)+\log_3(x+2)-\log_3\left(x^2-4\right)$

**12.** Given $\log_B(C)=3.1$, $\log_B(D)=4.2$, and $\log_B(E)=5.3$, evaluate $\log_B\left(\frac{CD}{E^2}\right)^3$.

**13.** Simplify. $\frac{3}{2}\log_2(x+y)+\log_2(x-y)-\log_2(x^2-y^2)$

**14.** Simplify. $\log_{12}(9)+\log_{12}(16x)-2\log_{12}(x)$

**15.** Simplify. $3\log_5(x-1)-2\log_5(x^2-1)+\log_5(2x^2+5x+3)$

**Week 6, Day 2**
**Lesson 5.2.4 Graphing Logarithmic Functions**
Students will solve logarithmic equations.

**1.** Solve $8^{x} = 4^{(2-x)}$.

**2.** Solve $3^{3x-1} = 243$.

**3.** Solve $32^{2x-1} = \left(\frac{1}{16}\right)^{-2x}$.

**4.** Solve $16^{x} \cdot \left(\frac{1}{32}\right)^{2-x} = 8$.

**5.** Solve $3\left(\frac{1}{3}\right)^{x-2} + 5 = 86$.

**6.** Solve $(3x)^{1/4} = 2$.

**7.** Solve $20m^{1.5} = 1000$.

**8.** Solve: $5x^{2/5} = 125$

**9.** Solve: $3x^{3.2} - 3 = 570$

**10.** Solve: $5(4x)^{5.5} - 1 = 54$

**11.** Solve: $5(2)^{x} = 50$

**12.** Solve: $200\left(\frac{3}{2}\right)^{x+2} = 700$

**13.** Solve: $8(10)^{x} + 2 = 26$

**14.** Solve: $7(3)^{x+1} + 2 = 58$

**Week 6, Day 3**
**Lesson 5.2.5** **Applications of Exponentials and Logarithms**
**Students will use exponential functions and logarithms to model and answer questions about everyday situations.**

**1.** Solve: $\log_x(16) = -4$

**2.** Solve: $\log_3(x) = -2$

**3.** Solve: $\frac{1}{2} = \log_x(5)$

**4.** Solve: $\log_3(9^5) = x$

**5.** Solve for $k$: $e^{kx} = m^x$
(Note that $e$ is the base of natural log and not a variable.)

**6.** Solve for x: $3x^{-1} + 5n^{-2} = 10$

**7.** Solve:

a. $3^2 \cdot 81^{3x} = \left(\frac{1}{27}\right)^{4x+1}$

b. $\log_4\left(\log_3(5x)\right) = \frac{1}{2}$

**8.** Solve:

a. $19(2)^{x+1} = 444$

b. $\log_2(x + 1) - \log_2(x - 2) = \log_3(9)$

**9.** Solve: $\log_3(81) - \log_5(125) = 4^{x-15}$

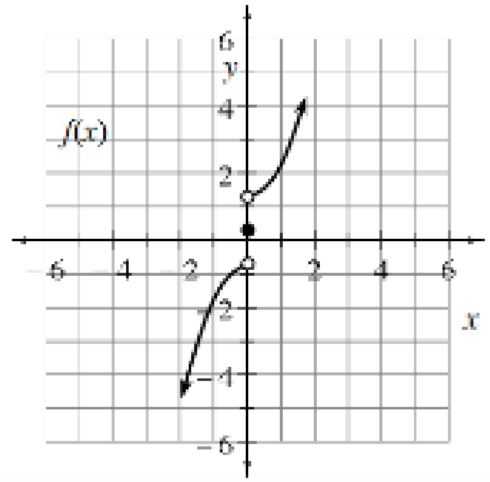
**10.** Solve.

a. $e^{4x} - 13e^{2x} + 12 = 0$

b. $\log_3(x - 1) + 5^{\log_5(2)} - \log_3(x) = 0$

**11.** Solve. a. $\ln(x - 2) - \ln(e + 5) = 1$

**Week 6, Day 4**
**Lesson 7.1.1 An Introduction to Limits**
**Students will be introduced to the concept of a limit in a geometric context.**




**1.** Use the graph of $y = f(x)$ at right to evaluate the following expressions.

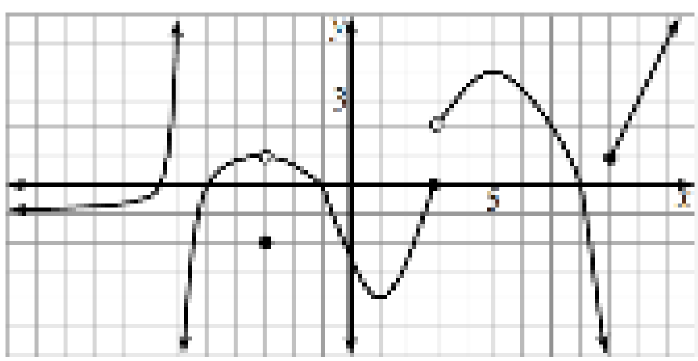
a. $f(0)$

b. $\lim_{x\to 0^-} f(x)$

c. $\lim_{x\to 0^+} f(x)$

d. $\lim_{x\to 0} f(x)$

**2.** Given the graph of $y = f(x)$, evaluate the following expressions.




a. $\lim_{x\to \infty} f(x)$

b. $\lim_{x\to -\infty} f(x)$

c. $\lim_{x\to 9^-} f(x)$

d. $\lim_{x\to 9^+} f(x)$

e. $f(-3)$

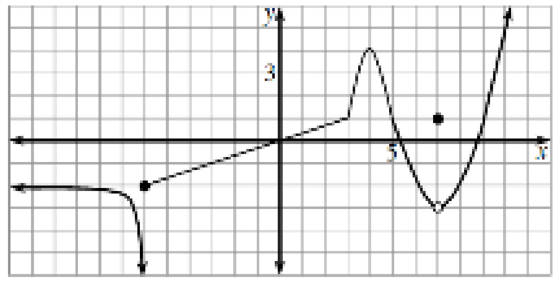
f. $\lim_{x\to -3} f(x)$

g. $f(3)$

h. $\lim_{x\to 3} f(x)$

**Week 7, Day 1**
**Lesson 7.1.2 Working with One Sided Limits**
**Students will work with one sided limits and limits at infinity**

**1.** Given the graph of $y = f(x)$, evaluate the following expressions.




a. $f(-6)$

b. $\lim_{x\to -6^-} f(x)$

c. $\lim_{x\to 3} f(x)$

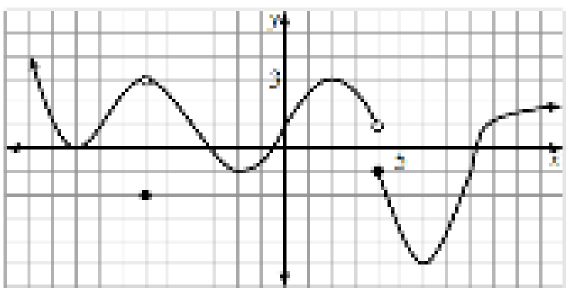
d. $f(7)$

e. $\lim_{x\to 7} f(x)$

f. $\lim_{x\to \infty} f(x)$

g. $\lim_{x\to -\infty} f(x)$

**2.** Given the graph of $y = f(x)$, evaluate the following expressions.




a. $f(-6)$

b. $\lim_{x\to -6} f(x)$

c. $\lim_{x\to 4^-} f(x)$

d. $\lim_{x\to 4^+} f(x)$

e. $f(4)$

f. $\lim_{x\to \infty}$

**3.** Sketch the graph of $f(x) = \begin{cases} x^2+5 & \text{for } x<1 \\ -2x+1 & \text{for } x\ge 1 \end{cases}$. Use it to evaluate the limit statements below.

a. $\lim_{x\to 3} f(x)$

b. $\lim_{x\to 1^-} f(x)$

c. $\lim_{x\to 1^+} f(x)$

**4.** Sketch the graph of $f(x) = \begin{cases} 4x+3 & \text{for } x<2 \\ x^2-3 & \text{for } x \geq 2 \end{cases}$. Use it to evaluate the limit statements below.

a. $\lim\limits_{x \to -1} f(x)$ b. $\lim\limits_{x \to 2^-} f(x)$ c. $\lim\limits_{x \to 2^+} f(x)$

**Week 7, Day 2**
**Lesson 7.1.3** The Definition of a Limit
**Students will analyze limits graphically and from tables.**

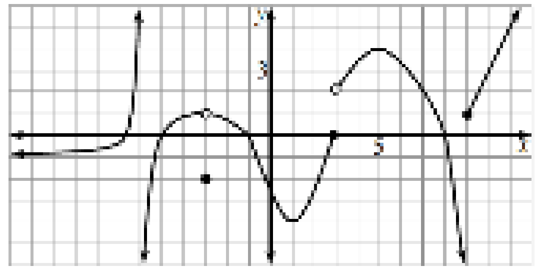
**1.** Given the graph of $y = f(x)$, evaluate the following expressions.

a. $\lim_{x \to \infty} f(x)$ b. $\lim_{x \to -\infty} f(x)$

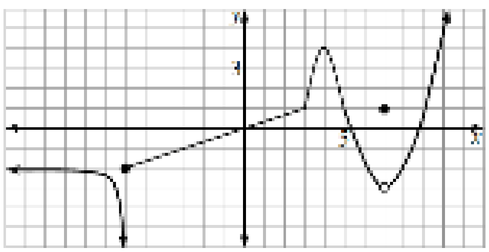
c. $\lim_{x \to 9^-} f(x)$ d. $\lim_{x \to 9^+} f(x)$

e. $f(-3)$ f. $\lim_{x \to -3} f(x)$

g. $f(3)$ h. $\lim_{x \to 3} f(x)$



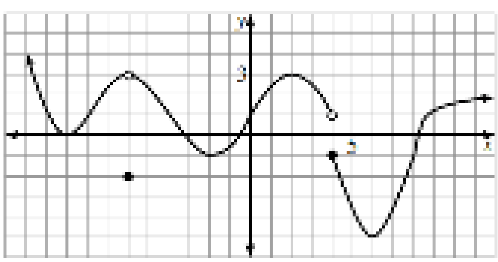
**2.** Given the graph of $y = f(x)$, evaluate the following expressions.



a. $f(-6)$ b. $\lim_{x \to -6^-} f(x)$ c. $\lim_{x \to 3} f(x)$ d. $f(7)$

e. $\lim_{x \to 7} f(x)$ f. $\lim_{x \to \infty} f(x)$ g. $\lim_{x \to -\infty} f(x)$

**3.** Given the graph of $y = f(x)$, evaluate the following expressions.



a. $f(-6)$ b. $\lim_{x \to -6} f(x)$ c. $\lim_{x \to 4^-} f(x)$ d. $\lim_{x \to 4^+}$
$f(x)$

e. $f(4)$ f. $\lim_{x \to \infty}$ g. $\lim_{x \to -\infty}$

**4.** Evaluate $\lim\limits_{h \to 0}(h^2+3h-4)$.

**5.** Let $f(x)=\dfrac{x^2+3x-10}{x-2}$. Complete the table of values below to predict the value of $\lim\limits_{x \to 2}\dfrac{x^2+3x-10}{x-2}$.

| $x$ | 1.9 | 1.99 | 1.999 | 2.001 | 2.01 | 2.1 |
|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | |

**6.** Let $f(x)=\dfrac{x^3-8}{x-2}$. Complete the table of values below to predict the value of $\lim\limits_{x \to 2} = \dfrac{x^3-8}{x-2}$.

| $x$ | 1.9 | 1.99 | 1.999 | 2.001 | 2.01 | 2.1 |
|---|---|---|---|---|---|---|
| $f(x)$ | | | | | | |

**7.** Use a graph or a table to evaluate $\lim\limits_{x \to \infty}\left(\dfrac{1}{x-2}+5\right)$.

**8.** Use a graph or a table to evaluate $\lim\limits_{x \to 4}\dfrac{x^2-6}{x+1}$.

**9.** Use a graph or a table to evaluate $\lim\limits_{x \to 5^-}\left(\dfrac{2}{x-5}+1\right)$.

**10.** Use a graph or a table to evaluate $\lim\limits_{x \to \infty}\dfrac{1}{2x-1}+3$.

**11.** Use a graph or a table to evaluate $\lim\limits_{x \to -2}\dfrac{x^2+1}{x+1}$.

**12.** Use a graph or a table to evaluate $\lim\limits_{x \to -2}\dfrac{x^2+1}{x+1}$.

**13.** Use a graph or a table to evaluate $\lim\limits_{x \to 1^+}\left(\dfrac{3}{x-1}-2\right)$.

**14.** Let $f(x)=\begin{cases} 2^x-1 & \text{for } x\le 3 \\ 2x+1 & \text{for } x>3 \end{cases}$.

a. Evaluate $\lim\limits_{x\to 3} f(x)$.

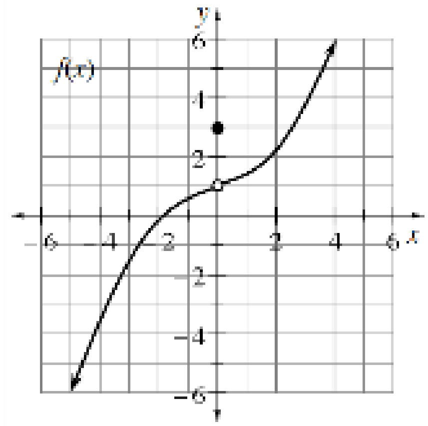
b. Evaluate $\lim\limits_{x\to 0} f(x)$.

**15.** Let $f(x)=\begin{cases} x^3+2x-1 & \text{for } x\le 1 \\ \dfrac{-4x+7}{x+1} & \text{for } x>1 \end{cases}$.

a. Evaluate $\lim\limits_{x\to 1} f(x)$.

b. Evaluate $\lim\limits_{x\to 2} f(x)$.

**Week 7, Day 3**
**Lesson 7.1.4 Limits and Continuity**
**Students will apply the formal definition of continuity.**

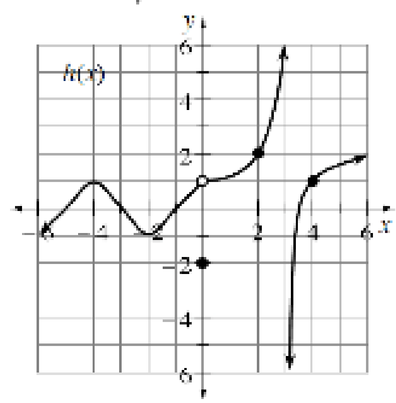



**1.** Use the graph at right to evaluate the following expressions.

a. $f(0)$

b. $\lim_{x \to 0} f(x)$

c. Is the function is continuous at x = 0? Explain your answer using the formal definition of continuity.




**2.** Use the graph of $y = h(x)$ below to complete the parts below. Explain each of your answers using the formal definition of continuity.

a. Is the function is continuous at x = 0?

b. Is the function is continuous at x = 2?

c. Is the function is continuous at x = 3?

**3.** Determine whether the given function is continuous at the given point. Explain your answer using the formal definition of continuity.

$$f(x) = 3^{x+2} - 1 \quad \text{at } x = -2$$

**4.** Determine whether the given function is continuous at the given point. Explain your answer using the formal definition of continuity.

$$f(x) = \frac{2x-1}{x-4} + 3 \quad \text{at } x = 4$$

**5.** Determine whether the given function is continuous at the given point. Explain your answer using the formal definition of continuity.

$$f(x) = \frac{x^2-25}{x+5} + 1 \quad \text{at } x = -5$$

**6.** Determine whether the given function is continuous at the given point. Explain your answer using the formal definition of continuity.

$$f(x)=\begin{cases} x^2-1 & \text{for } <3 \\ 2(x+1) & \text{for } x\ \geq 3 \end{cases} \quad \text{at } x = 3$$

**Week 7, Day 4**
**Lesson 7.2.1 Special Limits**
**Students will calculate average rates of change by calculating the slope of the secant line between two data points**.

**1.** The amount of money in a bank account at the end of each month is given in the table below.

| time (months) | 0 | 1 | 3 | 6 | 9 | 12 |
|---|---|---|---|---|---|---|
| balance ($) | 507 | 1276 | 1104 | 1353 | 987 | 1074 |

a. Graph the data.

b. Calculate the average change in the balance for each time interval ( 0 to 1, 2 to 3, …, 9 to 12 ). How does this relate to the graph?

c. Is the average rate of change for the entire 12 months increasing or decreasing? By how much?

d. Assuming the account balance continues in a similar pattern, how much money would you expect to be in the account after 10 years?

**2.** The table below shows the amount of carbon emissions (in million tons) worldwide for the given years. Enter the data into your calculator and create a plot of the data.

| Year | Emissions | Year | Emissions | Year | Emissions |
|---|---|---|---|---|---|
| 1950 | 1620 | 1975 | 4527 | 1992 | 5926 |
| 1955 | 2020 | 1980 | 5170 | 1993 | 5919 |
| 1960 | 2543 | 1985 | 5286 | 1994 | 5989 |
| 1965 | 3095 | 1988 | 5809 | 1995 | 6080 |
| 1970 | 4006 | 1990 | 5943 | | |

a. Write an equation to model the data in the plot.

b. What is the average rate of growth between 1950 and 1995?

c. Use your model to predict the amount of carbon emissions in 2020 and the rate at which the emissions will be changing near that time.

**3.** The amount of money in a bank account at the end of each month is given in the table below.

| time (months) | 0 | 1 | 3 | 6 | 9 | 12 |
|---|---|---|---|---|---|---|
| balance ($) | 507 | 1276 | 1104 | 1353 | 987 | 1074 |

a. Graph the data.

b. Calculate the average change in the balance for each time interval ( 0 to 1, 2 to 3, …, 9 to 12 ). How does this relate to the graph?

c. Is the average rate of change for the entire 12 months increasing or decreasing? By how much?

d. Assuming the account balance continues in a similar pattern, how much money would you expect to be in the account after 10 years?

**4.** The table below shows the amount of carbon emissions (in million tons) worldwide for the given years. Enter the data into your calculator and create a plot of the data.

| Year | Emissions | Year | Emissions | Year | Emissions |
|---|---|---|---|---|---|
| 1950 | 1620 | 1975 | 4527 | 1992 | 5926 |
| 1955 | 2020 | 1980 | 5170 | 1993 | 5919 |
| 1960 | 2543 | 1985 | 5286 | 1994 | 5989 |
| 1965 | 3095 | 1988 | 5809 | 1995 | 6080 |
| 1970 | 4006 | 1990 | 5943 | | |

a. Write an equation to model the data in the plot.

b. What is the average rate of growth between 1950 and 1995?

c. Use your model to predict the amount of carbon emissions in 2020 and the rate at which the emissions will be changing near that time.

**Week 8, Day 1**
**Lesson 7.2.2 Slope and Rates of Change**
**Students will calculate average rates of change from an equation of a function.**

**1.** A rocket is launched off of a platform such that its height is determined by the function $h(t) = -16t^2 + 128t + 4$, where $t$ is time in seconds.

a. When will the rocket hit the ground?

b. Make a table of time versus height. Use 1-second increments. Use the table to sketch a graph of the function over the interval that fits this situation.

c. What are the average velocities for each 1-second time interval that the ball is in the air?

d. What is happening to the average velocity of the ball with respect to the time?

e. What does the average velocity tell you about the change in position of the ball?

**2.** A man standing on a bridge drops a coin into a water fountain from a height of 105 ft. The height of the coin with respect to time is given by the function $h(t) = 105 - 16t^2$, where $t$ is in seconds and $t \geq 0$.

a. Calculate the average rate of change of the coin for the first 2 seconds after it is dropped. Include units in your answer.

b. What is the average rate of change of the coin between 2 seconds and the time it takes to hit the water?

**3.** Let $f(x) = \frac{1}{x+10} - 10$. Calculate the average rate of change from x = 2 to x = 4.

**4.** Let $f(x) = \sqrt{x+8} - 7$. Calculate the average rate of change from x = 4 to x = 10.

**5.** Let $f(x) = 3\sqrt{x} - 5$.

a. Calculate the average rate of change of $f$ from x = 3 to x = 4.

b. Write an expression for the average rate of change of $f$ from x = 9 to x = 9 + $h$.

c. Evaluate the expression in part (b) as $h \to 0$. What does this tell you about the function at x = 9?

**Week 8, Day 2**
**Lesson 7.2.3 Average Velocity and Rates of Change**
**Students will calculate average rates of change on smaller and smaller intervals.**

**1.** Write an expression for the average rate of change for the function $f(x) = 2x^2 - x$ between $x = 3$ and $x = 3 + h$. Simplify your answer completely.

**2.** Write an expression for the average rate of change for the function $f(x) = x^2 + 6x$ between $x = 2$ and $x = 2 + h$. Simplify your answer completely.

**3.** Write an expression for the average rate of change for the function $f(x) = x^2 - 4$ between $x = 5$ and $x = 5 + h$. Simplify your answer completely.

**4.** Write an expression for the average rate of change for the function $f(x) = 3x^2 + 4x$ between $x = 2$ and $x = 2 + h$. Simplify your answer completely.

**Week 8, Day 3**
**Lesson 7.2.4: Moving from AROC to IROC**
**Students will use the limit as $h \to 0$ for the average rate of change to calculate the instantaneous rate of change.**

**1.** Write and simplify an expression for the average rate of change for the function $g(x) = -4x^2 + 3x - 9$ between $x = -4$ and $x = -4 + h$.

**2.** Write and simplify an expression for the average rate of change for the function $h(x) = -7x^2 + 5x - 4$ between $x = 8$ and $x = 8 + h$.

3. Write and simplify an expression for the average rate of change for the function $j(x) = \frac{5}{x} + 8$ between $x = -1$ and $x = -1 + h$.

**4.** Write and simplify an expression for the average rate of change for the function $k(x) = -\frac{3}{x-8} - 1$ between $x = 9$ and $x = 9 + h$.

**Week 8, Day 4**
**Lesson 7.2.5 Rate of Change Applications**
**Students will use the limit as *h*→*0* for the average rate of change to calculate the instantaneous rate of change.**

**1.** Let $f(x) = 3\sqrt{x} - 5$.

a. Calculate the average rate of change of $f$ from x = 3 to x = 4.

b. Write an expression for the average rate of change of $f$ from x = 9 to x = 9 + $h$.

c. Evaluate the expression in part (b) as $h \to 0$. What does this tell you about the function at x = 9?

**2.** Let $f(x) = \frac{1}{3x} - x$.

a. Write and simplify an expression for the average rate of change of $f$ from x = 7 to x = 7 + $h$.

b. Evaluate the expression in part (a) as $h \to 0$. What does this tell you about the function at x = 7?

**3.** Let $f(x) = x^2 + x + 5$.

a. Write and simplify an expression for the average rate of change of $f$ from x = $a$ to x = $a + h$.

b. Evaluate the expression in part (a) as $h \to 0$. What does this tell you about the function at x = $a$?

**4.** Let $f(x) = 3x^2 - 2x - 4$.

a. Write and simplify an expression for the average rate of change of $f$ from x = $a$ to x = $a + h$.

b. Evaluate the expression in part (a) as $h \to 0$. What does this tell you about the function at x = $a$?

**Week 9, Day 1**
**Lesson 8.1.1 Graphing Transformations of the Sine Function**
**Students will combine a horizontal stretch and shift of the same trigonometric function and set up a modeling problem.**

**1.** Graph two complete cycles of $y = 2\cos(4x) - 1$. State or label all key features of the graph.

**2.** Graph two complete cycles of $y = 2\cos\left(x - \frac{\pi}{4}\right) + 1$. State or label all key features of the graph.

**3.** Graph two complete cycles of $y = -3\sin\left(x + \frac{\pi}{2}\right)$. State or label all key features of the graph. Then write an equivalent equation using cosine.

**4.** Graph two complete cycles of $y = -2\cos\left(x - \frac{\pi}{3}\right) - 2$.

**5.** Graph two complete cycles of $y = 3\sin\left(\frac{x}{3}\right) - 1$.

**Week 9, Day 2**
**Lesson 8.1.2 Modeling with Periodic Functions**
**Students will graph complex trigonometric functions.**

**1.** Graph two complete cycles of $y = 3\sin\left(\frac{\pi}{2}(x - 2)\right) + 1$.

**2.** Graph the given equation. $y = 2\cos(2(x - \pi)) + 1$

**3.** Graph the given equation. $y = -3\cos(2\pi(x + 1)) - 2$

**4.** Graph the given equation. $y = -2\sin\left(\frac{\pi}{2}(x + 2)\right) - 1$

**5.** Graph the given equation. $y = 2\sin\left(\frac{1}{2}\left(x + \frac{\pi}{3}\right)\right) + 1$

**Week 9, Day 3**
**Lesson 8.1.3 Improving the Spring Problem**
**Students will identify the amplitude and period of a trigonometric function and transform their graphs.**

**1.** Identify the amplitude, period, horizontal shift, and vertical shift of the given trigonometric function. Use the information to sketch two complete cycles of the function.

$$f(x) = 4 - 3\sin(2\pi x + \pi)$$

**2.** Identify the amplitude, period, horizontal shift, and vertical shift of the given trigonometric function. Use the information to sketch two complete cycles of the function.

$$f(x) = -3\cos(2x - 1) + 2$$

**3.** Identify the amplitude, period, horizontal shift, and vertical shift of the given trigonometric function. Use the information to sketch two complete cycles of the function.

$$f(x) = 2\sin(4x + \pi) - 1$$

**4.** Identify the amplitude, period, horizontal shift, and vertical shift of the given trigonometric function. Use the information to sketch two complete cycles of the function.

$$f(x) = 3 + 2\cos(3 - 6x)$$

**Week 9, Day 4**
**Lesson 8.2.1 Graphing Reciprocal Trigonometric Functions**
**Students will look at the values and graphs of all 6 trigonometric functions.**

**1.** If $\tan(\theta) = \frac{4}{3}$ and $\pi \le \theta \le \frac{3\pi}{2}$, determine the values of the other five trigonometric ratios.

**2.** If $\csc(\theta) = -\frac{7}{5}$ and $\frac{3\pi}{2} \le \theta \le 2\pi$, determine the values of the other five trigonometric ratios.

**3.** If $\sec(\theta) = \frac{8}{3}$ and $0 \le \theta \le \frac{\pi}{2}$, determine the values of the other five trigonometric ratios.

**4.** If $\cos(\theta) = \frac{9}{16}$ and $\frac{3\pi}{2} \le \theta \le 2\pi$, evaluate $\csc(\theta)$ and $\cot(\theta)$.

**5.** If $\tan(\theta) = -\frac{5}{12}$ and $\frac{3\pi}{2} \le \theta \le 2\pi$, determine the values of the other five trigonometric ratios.

**Week , Day 5**
**Lesson 8.3.1 Simplifying Trigonometric Expressions**
**Students will simplify trigonometric expressions by rewriting in terms of sine and cosine and determine special angles for trigonometric ratios from the unit circle.**

**1.** Rewrite the given expression as a single trigonometric ratio.

$\tan(A)(\csc(A) - \sin(A))$

**2.** Rewrite the given expression as a single trigonometric ratio.

$(\csc(x) + \cot(x))(1 - \cos(x))$

**3.** Simplify the following trigonometric expression. $\frac{\sec(x) - \cos(x)}{\tan(x)}$

**4.** Simplify the following trigonometric expression. $(\cos^2(x))(\sec^2(x) - 1)$

**5.** Simplify the following trigonometric expression. $\frac{1 + \sec(x)}{\sin(x) + \tan(x)}$

**6.** Simplify the following trigonometric expression. $\sin^2(x) \cdot \sec(x) + \cos(x)$

**Week 10, Day 1**
**Lesson 8.3.2 Proving Trigonometric Identities**

**Students will prove trigonometric identities.**

**1.** Simplify the following trigonometric expression. $\dfrac{1+\cos(x)}{1+\sec(x)}$

**2.** Simplify: $\dfrac{\sin(x)}{\csc(x)} - \sin(x)\csc(x)$

**3.** Simplify: cos(x) + tan(x) sin(x)

**4.** Let *f*(x) = csc(x) tan(x).

a. Graph *y* = *f*(x).

b. Evaluate $\lim\limits_{x\to 0} f(x)$.

c. Is *f* continuous at x = 0. Use the formal definition of continuity to explain your answer.

**5.** Let *f*(x) = $\dfrac{1-\cos^2(x)}{\sin(x)}$.

a. Graph *y* = *f*(x).

b. Evaluate $\lim\limits_{x\to\infty} f(x)$.

c. Use a table or graph to evaluate $\lim\limits_{x\to 0} f(x)$.

d. Is *f* continuous at x = 0. Use the formal definition of continuity to explain your answer.

**6.** Simplify: $\dfrac{1}{2}\left(\dfrac{1+\sin(x)}{\cos(x)} + \dfrac{\cos(x)}{1+\sin(x)}\right)$

**7.** Simplify: sec(x) − $\dfrac{\cos(x)}{1+\sin(x)}$ **8.** Simplify: sin(x)[cot(x) + tan(x)]

**9.** Simplify: csc(θ)(csc(θ) − sin(θ)) + $\dfrac{\sin(\theta)-\cos(\theta)}{\sin(\theta)}$ + cot(θ)

**10.** Simplify: $\dfrac{1+csc(A)}{sec(A)}$ − cot(*A*)

**Week 10, Day 2**
**Lesson 8.3.3 Angle Sum and Difference Identities**
**Students will verify trigonometric identities.**

**1.** Verify the given trigonometric identity. $(\tan(x) + \cot(x))^2 = \sec^2(x) + \csc^2(x)$

**2.** Verify the given trigonometric identity.

$$\frac{\sin(x)}{\sec^2(x) - 1} = \frac{1 - \sin^2(x)}{\sin(x)}$$

**3.** Verify the given trigonometric identity.

$$(\tan(x) + 1)^2 = \frac{1 + 2\sin(x)\cos(x)}{\cos^2(x)}$$

**4.** Verify the given trigonometric identity. $\dfrac{\sin(x)}{1 - \cos(x)} + \dfrac{\sin(x)}{1 + \cos(x)} = 2\csc(x)$

**5.** Verify the trigonometric identity. $\dfrac{\cos(\theta)}{\sec(\theta) + \tan(\theta)} = 1 - \sin(\theta)$

**6.** Solve $\sin(x) - \sin(2x) = 0$ for all values of x. Show all work.

**7.** Use at least one of trigonometric identities you have learned to simplify or expand the given equation and then solve for x, given $0 \le x < 2\pi$.

$$\sin(2x) - \sin(x) = 0$$

**8.** Use at least one of trigonometric identities you have learned to simplify or expand the given equation and then solve for x, given $0 \le x < 2\pi$.

**Week 10, Day 3**
**Lesson 8.3.4 Double-Angle and Half-Angle Identities**
Students will use the angle sum and difference identities **and double-angle and half-angle identities for sine and cosine** to solve problems.

**1.** Use an angle sum or difference formula to determine the exact value of the given expression.

$$\sin(165°)$$

**2.** Use an angle sum or difference formula to determine the exact value of the given expression.

$$\tan(-15°)$$

**3.** Use an angle sum or difference formula to simplify the given expression and determine its exact value.

$$\cos(12°)\cos(18°) - \sin(12°)\sin(18°)$$

**4.** Use an angle sum or difference formula to simplify the given expression and determine its exact value.

$$\sin(74°)\cos(14°) - \cos(74°)\sin(14°)$$

**Week 10, Day 4**
**Lesson 8.3.5 Solving Complex Trigonometric Equations**
**Students will solve trigonometric equations using identities and algebraic simplifications.**

**1.** Solve $2\cos^2(x) + \sin(x) = 2$ for $0 \le x < 2\pi$. Show all work.

**2.** Solve $\sin(x) - \sin(2x) = 0$ for all values of x. Show all work.

**3.** Determine at least three values for x which satisfy the following equation: $\sin^2(x) - \sin(x) = 2$

**4.** Determine the solutions to the given trigonometric equation for $0 \le x \le 2\pi$.

$$\sin^2(x) - \sin(x) = \cos^2(x)$$

**5.** Determine the solutions to the given trigonometric equation for $0 \le x \le 2\pi$.

$$\tan(x)\cos(x) = \cos(x)$$

**6.** Determine the solutions to the given trigonometric equation for $0 \le x \le 2\pi$.

$$\sin^2(x) + \cos(x) + 1 = 0$$

**7.** Determine the solutions to the given trigonometric equation for $0 \le x \le 2\pi$.

$$2\sin(x)\cos(x) = -\sin(x)$$

# Selected Answers

## Lesson 3.1.1

1. a. $\dfrac{x^2+16x+33}{5(x+2)}$ b. $x(x-1)$

2. a. $\dfrac{-x-30}{3(2x+3)}$ b. $\dfrac{a}{3(a-2)(a+3)}$

3. a. $\dfrac{(x-1)}{3(x-5)}$ b. $\dfrac{14x}{(x-3)(x+4)}$

4. $\dfrac{10x+4y}{x^2-y^2}$ 5. $\dfrac{x^2-2x-5}{(x+2)(x-2)(x+1)}$ 6. $\dfrac{2x+1}{x+3}$ 7. $\dfrac{x^2-x-10}{(x+3)(x-3)(x+1)}$

8. $\dfrac{x^2-7x+75}{(x+8)(x-4)(x-7)}$

## Lesson 3.1.2

1. $\dfrac{y^2+x^2y^4}{x^4y^2-x^2}$ 2. $\dfrac{x^3+xy^2}{y^3+x^2y^4}$ 3. xy 4. $\dfrac{y^6+x^6}{x^3y^2(y-x)}$

5. $\dfrac{4x^2+x}{2x-1}$ 6. $-\dfrac{y+x}{3y+2x}$

## Lesson 3.1.3

1. $(-3,-1)$, $(-1,-3)$, $(1,3)$, $(3,1)$

2. $\left(\sqrt{10},\dfrac{6\sqrt{10}}{5}\right)$, $\left(-\sqrt{10},-\dfrac{6\sqrt{10}}{5}\right)$, $(3\sqrt{2},2\sqrt{2})$, $(-3\sqrt{2},-2\sqrt{2})$

3. $(\pm 9,3)$, $(\pm 27,-15)$

4. $\left(-4,\dfrac{3}{4}\right)$

## Lesson 3.1.4

1. $x^2 - 2x - 3$

2. $6x^4 + 5x^3 + 5x^2 + 5x + 5$

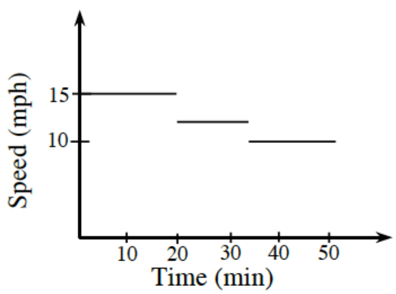
3. $x^3 + x^2 + 2x - 3 - \frac{2}{x-1}$

4. Yes, it divides with a remainder of 0.

5. $\left[3x^3 - 6x^2 + 10x - 20 + \frac{39}{x+2}\right]$

## Lesson 3.1.5

1.

a. 


b. 11 miles

c. $\approx 12.45$ mph

2. 12, 16, 20

3. 25.6 feet

4. $42,222.22

5. $\frac{4x^2}{25}\pi$

## Lesson 3.2.1

1. 31 + 107 + 255 + 499

2. 8 + 17 + 32 + 53

3. 5 + 9 + 13 + 17 + 21

4. $\sum_{j=0}^{4} 0.4\left(\frac{1}{0.4j+2}\right)$ Other answers are possible.

5. $\sum_{j=0}^{9} 0.2\left(4^{0.2j+3}\right)$ Other answers are possible.

## Lesson 3.2.2

1. $\left[0.5\sum_{n=1}^{4}(0.5n)^2 = \sum_{n=1}^{4} 0.125n^2 = 3.75\text{un}^2\right]$

2. $\left[\sum_{n=1}^{6} 0.5\left(\frac{4}{0.5n+1}\right) \approx 4.871\text{u}^2\right]$

3. $\left[\sum_{n=0}^{4}\left(0.6\left(2(0.6n+1)^2\right)\right) = 33.36\right]$

4. $\left[\sum_{n=0}^{9}\left[0.2\left(\sqrt{0.2n+5-2}\right)\right] \approx 3.939\text{u}^2\right]$

## Lesson 3.2.3

1. $\left[\sum_{n=1}^{20}\left[0.3\left((0.3\text{n}+4)^2 + (0.3\text{n}+4) - 12\right)\right]\right]$

2. $\sum_{n=1}^{8}\frac{3}{8}\left(\frac{\frac{3}{8}n+2+1}{\frac{3}{8}n+2-2}\right) = \frac{3}{8}\sum_{n=1}^{8}\left(1+\frac{8}{n}\right) \approx 11.154\text{u}^2$

3. $0.5\sum_{n=0}^{6}\left(-2(0.5n-1)^2+(0.5n-1)+6\right)=14\text{u}^2$

4. $0.1\sum_{n=0}^{29}\frac{3(0.1n-3)}{2(0.1n-3)-1}\approx 3.102\text{u}^2$

## Lesson 3.2.4

1. $\sum_{n=1}^{5}[0.6(\frac{1}{3}(0.6n+1-1)^2+4)] = \sum_{n=1}^{5}[0.6(0.12n^2+4)] = 15.96\ \text{u}^2$; overestimate

2.

a. $\sum_{n=0}^{6}\left(\frac{3}{7}\left(3\left(\frac{3}{7}n+1\right)^2-6\left(\frac{3}{7}n+1\right)\right)\right)\approx 12.450$

b. underestimate

c. Change the summation to be $\sum_{x=1}^{7}$.

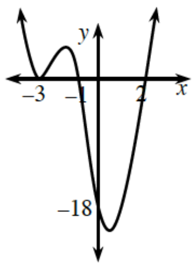
3. $[\sum_{n=1}^{5}(\frac{1}{1.2n-2}+7)\approx 37.482\ \text{un}^2]$

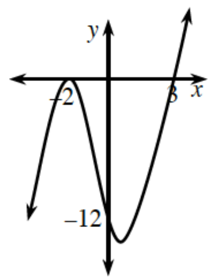
4. $\sum_{n=0}^{9}\left(5\sqrt{0.3n}+9\right)\approx 37.128\ \text{un}^2$

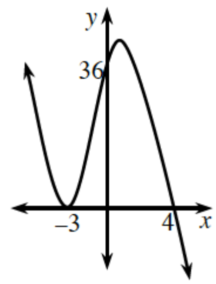
## U-Substitution

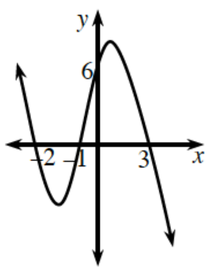
1. $(x+y-2)^2$
2. $4(a-b-1)^2$
3. $m(n+q)(n+q-2m)$
4. $(x^3-8)(x^3+1)$
5. $(x^5+4)(x^5-3)$
6. $(x^4+8)(x^4-6)$
7. $x=0,7$
8. $x=\frac{1}{3},\frac{1}{6}$
9. $x=-2,-1,1,2$

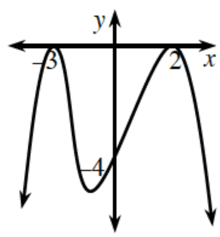
## Lesson 4.1.1

1. 


2. 


3. 


4. 


5. 


6. The graph decreases and flattens at (1, 0) and then continues to decrease for another 4.5 miles until it reaches the lowest point, approximately 3.4 miles below the ocean basin. The graph then starts to increase until it reaches the ocean basin again at (7, 0).

## Lesson 4.1.2

1. $p(x) = \frac{1}{2}(x^2 - 6x + 4)(x + 1)$    2. $p(x) = \frac{5}{4}(x^2 - 4)(x - 5)$    3. $p(x) = 3(x^2 + 2x - 2)(x - 4)$

4. $p(x) = a(x^2 - 14x + 53)(x + 5)(x - 3)^2$    5. $p(x) = \frac{1}{16}(x^2 - 6x + 13)(x^2 - 2x - 4)(x - 8)$

## Lesson 4.1.3

1. $x = 0,\ -2,\ \frac{4}{3}$    2. $x = \frac{1 \pm 2i\sqrt{5}}{3}$

3. $x = 0,\ \frac{-1 \pm 5i}{2}$

4. $x = 5,\ -1 \pm 3i$

5. a. No, $f(2) \neq 0$.

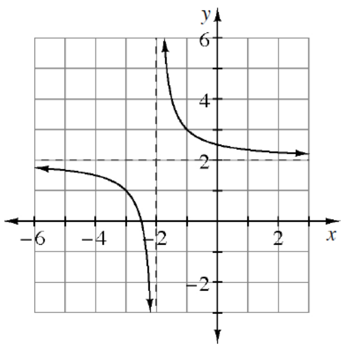
**b. The curve must cross the $x$-axis somewhere between those two points. More specifically, it must cross the $x$-axis somewhere in the interval of $-1.7 < x < 0$ because the $y$-intercept is $-4$.**

$x = -\frac{1}{2}, \frac{-1+\sqrt{17}}{2}$

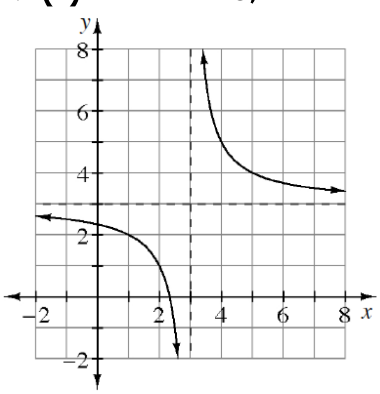
---

## Lesson 4.2.1

**1.** $f(x) = \frac{1}{x+2} + 2$;



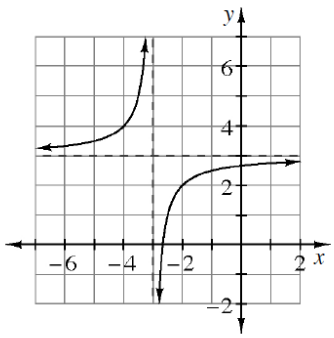

**2.** $f(x) = \frac{2}{x-3} + 3$;




**3.** $f(x) = \frac{a}{x+2} + 13$

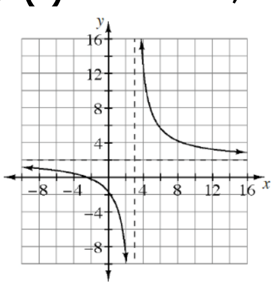
**4.** $f(x) = \frac{a}{x-9} - 12$

**5.** $f(x) = -\frac{1}{x+3} + 3$;

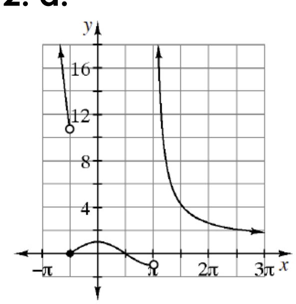



## Lesson 4.2.2

**1.** $f(x) = \dfrac{11}{x-3} + 2$;




**2. a.**




**b. D:** $x \neq \pi, 6$; **R:** $y \geq -4$

**3. a.** $n = -2$ **b.** $f(x) = -\dfrac{x+10}{x+3}$

**4.** $r(x) = 1 + \dfrac{b-a}{x-b}$; **If** $b = 0$**, there is no** $y$**-intercept. For** $a \neq b$**, no real values exist for** $a$ **such that there is no** $x$ **intercept. If** $a = b$**, then there is no** $x$**intercept**

**because $r(x) = 1$, which is a horizontal line (with a hole at $x = a$).**

**5. Allow the use of a graphing calculator on this problem.**

$$r(x) = \frac{4(x+1)^2(x-3)}{(x+1)(x+3)(x-4)(x-1)^2}\ ]$$

**6. Allow the use of a graphing calculator on this problem.**

$$r(x) = \frac{-15(x+2)(x-4)}{(x-4)(x+6)(x-5)}$$

---

**Lesson 4.2.3**

**1.** $y = 54$ **2.** $y = -1$ **3.** $y = 0$ **4.** $y = 0$ **5.** $y = \frac{3}{2}$

---

**Lesson 4.3.1**

**1.** $(-\infty, -6) \cup (2, \infty)$ **2.** $\left[-\frac{3}{2}, -\frac{1}{3}\right]$ **3.** $(-\infty, -1) \cup (5, 7)$

**4.** $(-\infty, -10) \cup (-6, 1]$

**5.** $[0, 6] \cup [10, \infty)$ **6.** $(-\infty, 0) \cup (0, 10)$ **7.** $(-\infty, -3) \cup [-2, \infty)$

---

**Lesson 4.3.2**

**1.** $x = -7;\ y = 2$ **2.** $x = 5;\ y = -5$

**3. slant:** $y = x^2 + 3x + 1$**; vertical:** $x = -2$ **4.** $x = -\frac{2}{3};\ y = \frac{5}{3}$

**5. a.** $t$ **= 0, 2, 5, 8**

**b. (0, 2) ∪ (2, 5) ∪ (8, 10)**

**c. 1.75 feet below the branch.**

## Lesson 5.1.1

1. $y = 16\left(\frac{3}{2}\right)^x$     2. $y = \frac{2}{3}(3)^x$     3. $y = 27\left(\frac{4}{3}\right)^x$

4. a. 42.67 g ]   [ b. After ≈ 27 hours.

5. 16.2 years

6. 1.24%

7. 6.23 years

## Lesson 5.1.2

1. $y = \frac{4}{81}(3)^x$     2. $y = 36(9)^x$     3. $y = \frac{5}{4}(8)^x$     4. $\left(\frac{5}{81}\right)(9)^x$     5. $y = 5(8^x)$

## Lesson 5.1.3

1. a. $2^{-3} = \frac{1}{8}$  b. $\log_A(B) = y$     2. a. $\log_5\left(\frac{1}{25}\right) = -2$     b. $B^K = M$

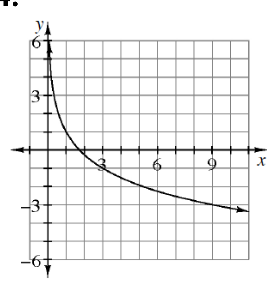
3. a. $V^C = I$     b. $\log_7(M) = x$

4. a. $f^{d-2} = g$     b. $\log_4(n+1) = x$     5. $f^{-1}(x) = 3^x - 2$

## Lesson 5.2.1

1. $f^{-1}(x) = \log_5\left(\frac{x+3}{4}\right)$     2. $f^{-1}(x) = \log_4\left(\frac{x-5}{3}\right)$     3. $f^{-1}(x) = 7^{\left(\frac{x+8}{6}\right)}$

4. $f^{-1}(x) = 3^{\left(\frac{x-1}{-2}\right)} = \left(\frac{\sqrt{3}}{3}\right)^{x-1}$ ;



5. a. $-\frac{1}{2}$ b. $f(x) = \log_2(x + 2)$

6. a. exponential (with base $e$) b. $f(x)=e^{-1.5x+1}=\frac{e}{e^{1.5x}}$

7. $f(x)=\sqrt{2^x}$; Methods will vary, but may include using a composition of functions to show that the end output is equal to the initial output, graphing the functions to show that they are symmetric with respect to the line $y = x$, the use of tables, and/or deriving the inverse equation using algebra.

8. Logarithms should be used to solve an equation that has the variable in the exponent.

9. Keith is correct. The variable is not in the exponent in this equation. $x=\left(\frac{9}{5}\right)^{3/2}=\frac{27\sqrt{5}}{25}$

---

Lesson 5.2.2

1. a. 4 b. −2 c. 5

2. a. 3 b. −2 c. −1

3. a. 5 b. 0.5 c. 0

4. a. 4 b. 1.25 c. 21

5. a. $-\frac{2}{3}$ b. $3x+3$

6. a. $-\frac{1}{2}$ b. 4

7. a. 1 b. $\sqrt{7}$

8. a. 16 b. $-6x$

9. a. $12x$ b. 320

10. a. $3x$ b. 2 ]

11. a. 71 b. $(x-1)^2$

12. a. $-x$ b. $-15x$

## Lesson 5.2.3

1. $\log\left(\frac{M^2}{N^3}\right)$

2. $2\log_a(x)-\log_a(y)-7\log_a(z)$

3. $\log_m(a)+2\log_m(b)+\frac{3}{2}\log_m(c)$

4. $\log_t(h)+\frac{1}{2}\log_t(h^2+g)$

5. 3.2

6. 16.2

7. −5.4

8. 15.9

9. $\log_7(4c^3d)$

10. $\log_7(3(x+6))$

11. $\log_3(x-2)$

12. −9.9

13. $\frac{1}{2}\log(x+y)$

14. $2-\log_{12}(x)$

15. $\log_5\left(\frac{(x-1)(2x+3)}{(x+1)}\right)$

## Lesson 5.2.4

1. x = $\frac{4}{5}$

2. x = 2

3. x = $\frac{5}{2}$

4. x = $\frac{13}{9}$

5. x = −1

6. x = $\frac{16}{3}$

7. x ≈ 13.572

8. x = $\pm 5^5$

9. x = $\pm 191^{5/16} \approx \pm 5.162$

10. x = $\frac{11^{2/11}}{4}$ ≈ 0.387

11. x = 3.32

12. x = 1.09

13. x = log(3) ≈ 0.477

14. x = 0.89

## Lesson 5.2.5

1. x = $\frac{1}{2}$

2. x = $\frac{1}{9}$

3. x = 25

4. x = 10

5. $k$ = ln($m$)

6. x = $\frac{3n^2}{10n^2-5}$

7. a. $x = -\frac{5}{24}$ b. $x = \frac{9}{5}$

8. a. $x \approx 3.546$ b. $x = 3$ 9. $x = 15$

10. a. $x = \frac{\ln(12)}{2} \approx 1.242$ or $x = \frac{\ln(1)}{2} = 0$ b. $x = \frac{9}{8}$

11. a. $x = e^2 + 5e + 2 \approx 22.980$ b. $x = -\ln(7) \approx -1.946$

---

**Lesson 7.1.1**

1 a. 2 b. 1 c. 3 d. DNE

2. a. $\infty$ b. $-1$ c. DNE but $f(x) \to -\infty$ d. 1 e. $-2$ f. 1
g. 0 h. DNE

---

**Lesson 7.1.2**

1. a. $-2$ b. DNE but $f(x) \to -\infty$ c. 1 d. 1 e. $-3$
f. DNE but $f(x) \to \infty$ g. $-2$

2. a. $-2$ b. 3 c. 1 d. $-1$ e. $-1$ f. 2

g. DNE but $f(x) \to \infty$

3. a. $-5$ b. 6 c. $-1$

4. a. $-1$ b. 11 c. 1

---

**Lesson 7.1.3**

1. a. $\infty$ b. $-1$ c. DNE but $f(x) \to -\infty$ d. 1 e. $-2$

f. 1 g. 0 h. DNE

2. a. $-2$ b. DNE but $f(x) \to -\infty$ c. 1 d. 1 e. $-3$

f. DNE but $f(x) \to \infty$ g. $-2$

3. a. $-2$ b. 3 c. 1 d. $-1$ e. $-1$ f. 2

g. DNE but $f(x) \to \infty$

4. –4

5. See completed table below. $\lim_{x \to 2} \frac{x^2+3x-10}{x-2} = 7$

| $x$ | 1.9 | 1.99 | 1.999 | 2.001 | 2.01 | 2.1 |
|---|---|---|---|---|---|---|
| $f(x)$ | 6.9 | 6.99 | 6.990 | 7.001 | 7.01 | 7.1 |

6. See completed table below. $\lim_{x \to 2} = \frac{x^3-8}{x-2} = 12$

| $x$ | 1.9 | 1.99 | 1.999 | 2.001 | 2.01 | 2.1 |
|---|---|---|---|---|---|---|
| $f(x)$ | 11.41 | 11.94 | 11.994 | 12.006 | 12.06 | 12.61 |

7. 5    8. 2    9. DNE, but the function $\to -\infty$    10. 3

11. –5    12. –5

13. DNE, but the function $\to \infty$    14. a. 7  b. 0

15. a. DNE  b. $-\frac{1}{3}$

---

Lesson 7.1.4

1.
a. 3,
b. 1,
c. not continuous; $\lim_{x \to 0} f(x) \neq f(0)$

2.
a. no; $\lim_{x \to 0} f(x) \neq f(0)$;
b. yes; $\lim_{x \to 2} f(x) = f(2)$;
c. no; $\lim_{x \to 3} f(x)$ DNE

3. continuous; $\lim_{x \to 2} f(x) = f(2)$

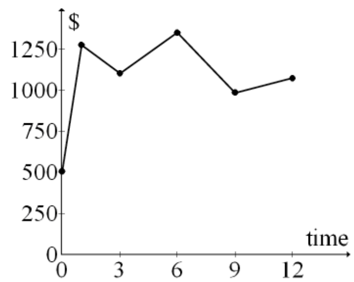
4. not continuous; $\lim\limits_{x \to 4} f(x)$ DNE

5. not continuous; $f(-5)$ DNE

6. continuous; $\lim\limits_{x \to 3^-} f(x) = \lim\limits_{x \to 3^+} f(x) = f(3)$

---

## Lesson 7.2.1

1. a. 


b.

| time period | change in account balance |
|---|---|
| 0 – 1 | 769 |
| 1 – 3 | –86 |
| 3 – 6 | 83 |
| 6 – 9 | –122 |
| 9 – 12 | 29 |

c. Increasing by approximately $47.25 per month.

d. $6177

2. a. Note: Students should use a calculator to create a model. $C(t) = 103.2t + 1677.3$ where $t$ is the number of years after 1950.

b. 99.11 million tons per year.

**c. 8901 million tons. The graph is linear, so the carbon emissions will still be increasing at a rate of approximately 103.2 million tons per year.]**

**3. b. See table below.**

| time interval (minutes) | change in HR (bpm) |
|---|---|
| 0 – 15 | 3.33 |
| 15 – 20 | 4 |
| 20 – 25 | 3 |
| 25 – 27 | 2.5 |
| 27 – 28 | −13 |
| 28 – 30 | −15 |

**c. At first, the as the heart rate increases, so does the change in heart rate. After 20 minutes the heart rate continues to increase, but the change in heart rate decreases. After 27 minutes the heart rate decreases and so does the change in heart rate. This is seen by the concavity of the graph. After 15 minutes the graph is concave down.**

**d. The person starts out jogging and their heart rate increases significantly. After 15 minutes the person starts to pick up speed. From approximately 25 to 27 minutes they run as fast as they can. They are then tired and go back to jogging for 3 more minutes until they are done with their workout.]**

---

**Lesson 7.2.2**

**1. a.** $t \approx$ **8.031 s**

**2. See table below.**

| $t$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| $h(t)$ | 4 | 116 | 196 | 244 | 260 | 244 | 196 | 116 | 4 | −140 |

c **See table below.**

| $\Delta t$ | 0 − 1 | 1 − 2 | 2 − 3 | 3 − 4 | 4 − 5 | 5 − 6 | 6 − 7 | 7 − 8 | 8 − 9 |
|---|---|---|---|---|---|---|---|---|---|
| $\Delta h(t)$ | 112 | 80 | 48 | 16 | −16 | −48 | −80 | −112 | −144 |

**d. It is decreasing by 32 over each interval. It changed from positive to negative after 4 seconds.**

**e. Positive velocity indicates that the ball is rising for 4 seconds. The velocity then changes to a negative value indicating that the ball is falling.**

**3. a. −32 ft/s**

**b. 2 − 2.56 seconds: AROC = −73 ft/s ]**

**c. Average velocity = 26.67 mph. Yes, at some point the car picked up speed and was traveling at 22.67 mph.**

**d. The car leaves home and drives through a neighborhood. After 5 minutes they realize that something was forgotten at home and must head back. After 10 minutes they arrive home. They head out again through the neighborhood. After 15 minutes they are on larger, faster city streets. After 30 minutes they get on a highway where they can travel at a higher rate of speed, but there is some traffic. ]**

**4.** $-\dfrac{1}{168}$

**5.** $\dfrac{3\sqrt{2}-2\sqrt{3}}{7}$

6. a. $6 - 3\sqrt{3}$  b. $\frac{3\sqrt{9+h}-9}{h} = \frac{3}{\sqrt{9+h}+3}$

c. $\frac{1}{2}$; This is the slope of the line tangent to the curve at $x = 9$. ]

## Lesson 7.2.3

1. $11 + 2h$  2. $10 + h$  3. $10 + h$  4. $3h + 16$ ]

## Lesson 7.2.4

1. $-4h + 35$  2. $-7h - 107$  3. $\frac{5}{-1+h}$  4. $\frac{3}{h+1}$

## Lesson 7.2.5

1. a. $6 - 3\sqrt{3}$  b. $\frac{3\sqrt{9+h}-9}{h} = \frac{3}{\sqrt{9+h}+3}$

c. $\frac{1}{2}$; This is the slope of the line tangent to the curve at $x = 9$. ]

2. a. $-\frac{1}{21(7+h)} - 1$

b. $-1\frac{1}{47}$; This is the slope of the line tangent to the curve at $x = 7$. ]

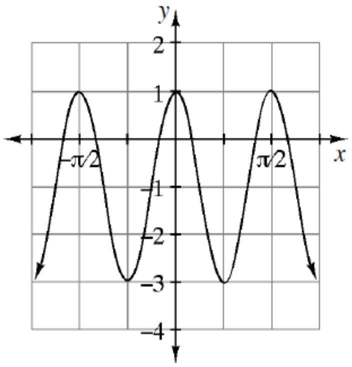
3. a. $h + 2a + 1$

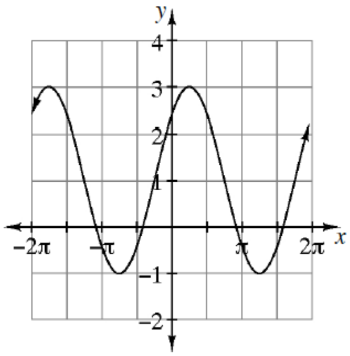
b. $2a + 1$; The slope of the line tangent to the curve at any point $x = a$ is $2a + 1$. ]

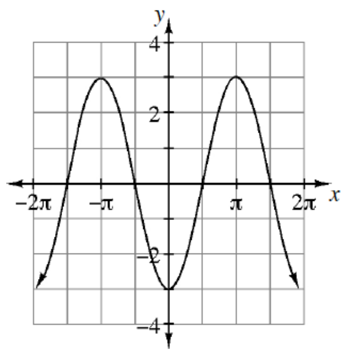
4. a. $3h + 6a - 2$

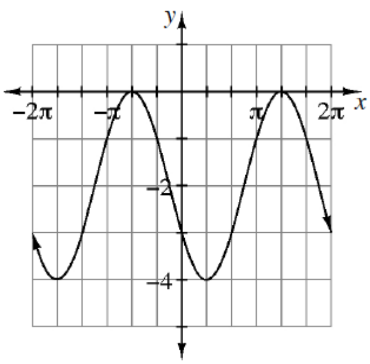
b. $6a - 2$; The slope of the line tangent to the curve at any point $x = a$ is $2a + 1$. ]

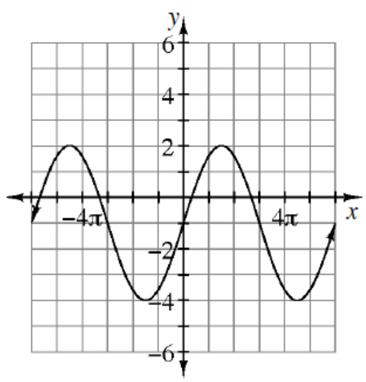
## Lesson 8.1.1

1. 

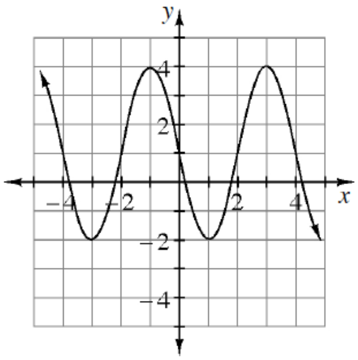

2. 

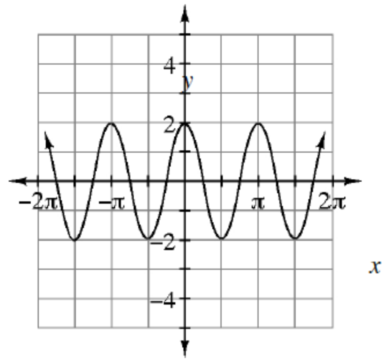

3. 

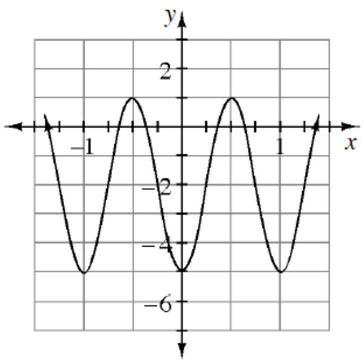

4. 

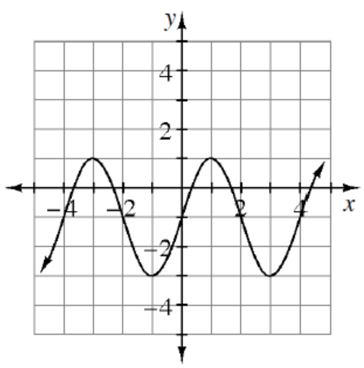

5. 

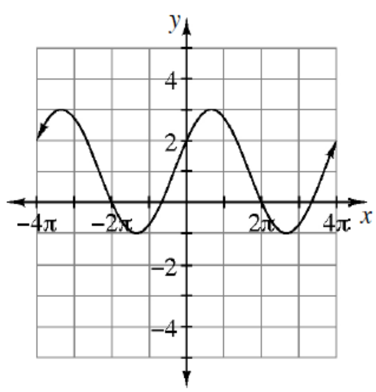
## Lesson 8.1.2

1. 


2. 

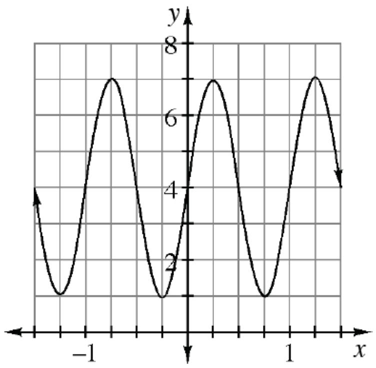

3. 


4. 

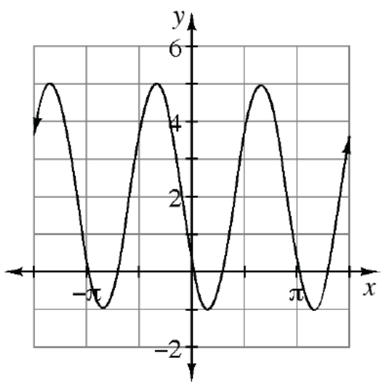

5. 

## Lesson 8.1.3

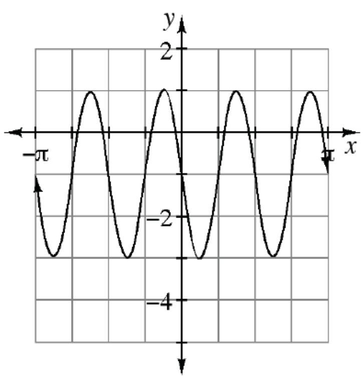
1. amplitude = 3, period = 1, horizontal shift = $-\frac{1}{2}$ units, vertical shift = 4 units.



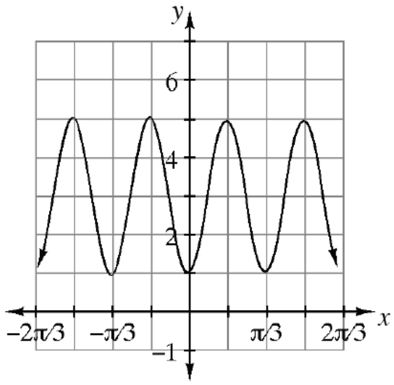

2. amplitude = 3, period = $\pi$, horizontal shift = $\frac{1}{2}$ units, vertical shift = 2 units.




3. amplitude = 2, period = $\frac{\pi}{2}$, horizontal shift = $-\frac{\pi}{4}$ units, vertical shift = –1 units.



**4. amplitude = 2, period = $\frac{\pi}{3}$ , horizontal shift = $\frac{1}{2}$ units, vertical shift = 3 units.**




**5. amplitude = 2, period = 8, vertically reflected, shifted right 3 units, shifted up 1 unit.**

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## Lesson 8.2.1

1. $\sin(\theta) = -\frac{4}{5}$, $\cos(\theta) = -\frac{3}{5}$, $\csc(\theta) = -\frac{5}{4}$, $\sec(\theta) = -\frac{5}{3}$, $\cot(\theta) = \frac{3}{4}$

2. $\sin(\theta) = -\frac{5}{7}$, $\cos(\theta) = \frac{2\sqrt{6}}{7}$, $\tan(\theta) = -\frac{5\sqrt{6}}{12}$, $\sec(\theta) = \frac{7\sqrt{6}}{12}$, $\cot(\theta) = -\frac{2\sqrt{6}}{5}$

3. $\sin(\theta) = \frac{\sqrt{55}}{8}$, $\cos(\theta) = \frac{3}{8}$, $\tan(\theta) = \frac{\sqrt{55}}{3}$, $\csc(\theta) = \frac{8\sqrt{55}}{55}$, $\cot(\theta) = \frac{3\sqrt{55}}{55}$

4. $\csc(\theta) = -\frac{16\sqrt{7}}{35}$, $\cot(\theta) = -\frac{9\sqrt{7}}{35}$

5. $\sin(\theta) = -\frac{5}{13}$, $\csc(\theta) = -\frac{13}{5}$, $\cos(\theta) = \frac{12}{13}$, $\sec(\theta) = \frac{13}{12}$, $\cot(\theta) = -\frac{12}{5}$

---

## Lesson 8.3.1

1. cos($A$) 2. sin($x$) 3. sin($x$) 4. $\sin^2(x)$

5. csc($x$) 6. sec($x$)

## Lesson 8.3.2

1. $\cos(x)$    2. $-\cos^2(x)$    3. $\sec(x)$

4. a. Teams should graph the function $y = \sec(x)$., b. 0, c. No, the limit exists, but $f(0)$ is undefined. ]

5. a. Teams should graph $y = \sin(x)$., b. DNE, c. 0, d. No, the limit exists, but $f(0)$ is undefined. ]

6. $\sec(x)$    7. $\tan(x)$    8. $\sec(x)$    9. $\csc^2(\theta)$

10. $\cos(A)$

## Lesson 8.3.3

1. $(\tan(x) + \cot(x))^2 = \sec^2(x) + \csc^2(x)$

$\tan^2(x) + 2\tan(x)\cot(x) + \cot^2(x) =$

$\tan^2(x) + 2 + \cot^2(x) =$

$\tan^2(x) + 1 + \cot^2(x) + 1 = \sec^2(x) + \csc^2(x)$ ]

2.

[ $\dfrac{\sin(x)}{\sec^2(x) - 1} = \dfrac{1 - \sin^2(x)}{\sin(x)}$

$\dfrac{\sin(x)}{\tan^2(x)} =$

$\sin(x) \cdot \dfrac{\cos^2(x)}{\sin^2(x)} =$

$\dfrac{\cos^2(x)}{\sin(x)} = \dfrac{1 - \sin^2(x)}{\sin(x)}$ ]

3.

[ $(\tan(x) + 1)^2 = \dfrac{1 + 2\sin(x)\cos(x)}{\cos^2(x)}$

$\tan^2(x) + 2\tan(x) + 1 =$

$\sec^2(x) + 2\tan(x) =$

$\dfrac{1}{\cos^2(x)} + \dfrac{2\sin(x)}{\cos(x)} = \dfrac{1 + 2\sin(x)\cos(x)}{\cos^2(x)}$ ]

4.

[ $\dfrac{\sin(x)}{1 - \cos(x)} + \dfrac{\sin(x)}{1 + \cos(x)} = 2\csc(x)$

$\dfrac{(\sin(x))(1 + \cos(x)) + (\sin(x))(1 - \cos(x))}{(1 - \cos(x))(1 + \cos(x))} =$

$\dfrac{\sin(x) + (\sin(x))(\cos(x)) + \sin(x) - (\sin(x))(\cos(x))}{1 + \cos(x) - \cos(x) - \cos^2(x)} =$

$\dfrac{2\sin(x)}{1 - \cos^2(x)} =$

$\dfrac{2\sin(x)}{\sin^2(x)} =$

$\dfrac{2}{\sin(x)} = 2\csc(x)$ ]

5.

[ $\dfrac{\cos(\theta)}{\sec(\theta) + \tan(\theta)} = 1 - \sin(\theta)$

$\dfrac{\cos(\theta)}{\dfrac{1}{\cos(\theta)} + \dfrac{\sin(\theta)}{\cos(\theta)}} =$

$$\frac{\frac{\cos(\theta)}{1+\sin(\theta)}}{\cos(\theta)} =$$

$$\cos(\theta) \cdot \frac{\cos(\theta)}{1+\sin(\theta)} =$$

$$\frac{\cos^2(\theta)}{1+\sin(\theta)} =$$

$$\frac{1-\sin^2(\theta)}{1+\sin(\theta)} =$$

$$\frac{(1+\sin(\theta))(1-\sin(\theta))}{(1+\sin(\theta))} = 1 - \sin(\theta)\ ]$$

6. $x = 0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$, all $+ 2\pi n$

7. $x = 0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$

---

## Lesson 8.3.4

1. $\frac{\sqrt{6}-\sqrt{2}}{4}$

2. $-2+\sqrt{3}$

3. $\cos(30°) = \frac{\sqrt{3}}{2}$

4. $\sin(60°) = \frac{\sqrt{3}}{2}$

5. $\frac{16}{65}$

---

## Lesson 8.3.5

1. $x = 0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}$

2. $x = 0, \pi, \frac{\pi}{3}, \frac{5\pi}{3}$, all $+ 2\pi n$

3. $\sin(x) = -1;\ x = \frac{3\pi}{2} + 2n\pi$

4. $x = \frac{7\pi}{6}, \frac{11\pi}{6}, \frac{\pi}{2}$

5. $x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{4}, \frac{5\pi}{4}$

6. $x = \frac{3\pi}{2}$

7. $x = 0, \pi, \frac{2\pi}{3}, \frac{4\pi}{3}$