Mathematics

Calculus

DETROIT
PUBLIC SCHOOLS
COMMUNITY DISTRICT

Letter to Families from the DPSCD Office of Mathematics

Dear DPSCD Families,

The Office of Mathematics is partnering with families to support Distance Learning while students are home. We empower you to utilize the resources provided to foster a deeper understanding of grade-level mathematics.

In this packet, you will find links to videos, links to online practice, and pencil-and-paper practice problems. The Table of Contents shows day-by-day lessons from April 14th to June 19th. We encourage you to take every advantage of the material in this packet.

Daily lesson guidance can be found in the table of contents below. Each day has been designed to provide you access to materials from Khan Academy and the academic packet. Each lesson has this structure:

Watch: Khan Academy (if internet access is available)	Practice: Khan Academy (if internet access is available)	Pencil & Paper Practice: Academic Packet
Watch and take notes on the lesson video on Khan Academy	Complete the practice exercises on Khan Academy	Complete the pencil and paper practice.

If one-on-one, live support is required, please feel free to call the **Homework Hotline** at 1-833-466-3978. Please check the Homework Hotline page for operating hours. We have DPSCD mathematics teachers standing by and are ready to assist.

We appreciate your continued dedication, support and partnership with Detroit Public Schools Community District and with your assistance we can press forward with our priority: Outstanding Achievement. Be safe. Be well!

Deputy Executive Director of K-12 Mathematics

Important Links and Information

Clever

Students access Clever by visiting www.clever.com/in/dpscd.

What are my username and password for Clever?
Students access Clever using their DPSCD login credentials. Usernames and passwords follow this structure:

Username: studentID@thedps.org
Ex. If Aretha Franklin is a DPSCD student with a student ID of 018765 her username would be 018765@thedps.org.

Password:
First letter of first name in upper case
First letter of last name in lower case
2-digit month of birth
2-digit year of birth
01 (male) or 02 (female)
For example: If Aretha Franklin's birthday is March 25, 1998, her password and password would be Af039802.

Accessing Khan Academy

To access Khan Academy, visit www.clever.com/in/dpscd. Once logged into Clever, select the Khan Academy button:

Accessing Your CPM eBook

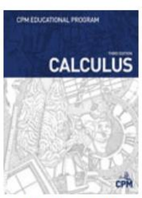
Students can access their CPM eBook in two ways:

Option 1: Access the eBook through Clever

1. Visit www.clever.com/in/dpscd. Login using your DPSCD credentials above.
2. Click on the CPM icon:

Option 2: Visit http://open-ebooks.cpm.org/

1. Visit the website listed above.
2. Click “I agree”
3. Select the CPM Calculus eBook:



Calculus Third Edition

Desmos Online Graphing Calculator

Access to a free online graphing and scientific calculator can be found at https://www.desmos.com/calculator.

Table of Contents

In the following table, you will find the table of contents and schedule for the week of April 13, 2020 through the week of June 15, 2020.

Week of 4/20-4/24	Day 1	Lesson 4.2.2: Indefinite and Definite Integrals	The Fundamental Theorem of Calculus and Accumulation Functions	The Fundamental Theorem of Calculus and Accumulation Functions	
	Day 2		Khan Academy Integration Quiz 1	Khan Academy Integration Quiz 1	
	Day 3	Lesson 4.2.3: The Fundamental Theorem of Calculus	The Fundamental Theorem of Calculus and Accumulation Functions	The Fundamental Theorem of Calculus and Accumulation Functions	Problems 1 - 4
	Day 4	Lesson 4.2.4: The Fundamental Theorem of Calculus	The Fundamental Theorem of Calculus and Accumulation Functions	The Fundamental Theorem of Calculus and Accumulation Functions	
	Day 5	Lesson 4.2.5: Integrals as Accumulators	The Fundamental Theorem of Calculus and Accumulation Functions	The Fundamental Theorem of Calculus and Accumulation Functions	Problems 1 - 4

Week of **4/27-05/01**	Day 1	Lesson 4.4.1: Area between Curves	Khan Academy Area between Curves Expressed as Functions of x Khan Academy Area between Curves Expressed as Functions of y Khan Academy Finding Area between Two Curves that Intersect at Multiple Points	Khan Academy Area between Curves Expressed as Functions of x Khan Academy Area between Curves Expressed as Functions of y Khan Academy Finding Area between Two Curves that Intersect at Multiple Points	
	Day 2	Lesson 4.4.2: More Area between Curves	Khan Academy Area between Curves Expressed as Functions of x Khan Academy Area between Curves Expressed as Functions of y Khan Academy Finding Area between Two Curves that Intersect at Multiple Points	Khan Academy Area between Curves Expressed as Functions of x Khan Academy Area between Curves Expressed as Functions of y Khan Academy Finding Area between Two Curves that Intersect at Multiple Points	

	Day 3	Lesson 4.4.3: Multiple Methods for Calculating Area between Curves	Khan Academy Area between Curves Expressed as Functions of x Khan Academy Area between Curves Expressed as Functions of y Khan Academy Finding Area between Two Curves that Intersect at Multiple Points	Khan Academy Area between Curves Expressed as Functions of x Khan Academy Area between Curves Expressed as Functions of y Khan Academy Finding Area between Two Curves that Intersect at Multiple Points	
	Day 4	Chapter 4 Closure	Integration as Accumulation and Change Quiz 2 Integration as Accumulation and Change Quiz 3	Integration as Accumulation and Change Quiz 2 Integration as Accumulation and Change Quiz 3	Chap 4 Review
	Day 5	Lesson 5.1.1: Distance, Velocity, and Acceleration Functions	Khan Academy Position, Velocity and Acceleration Khan Academy Connecting Position, Velocity, and Acceleration	Khan Academy Position, Velocity and Acceleration Khan Academy Connecting Position, Velocity, and Acceleration	Problems 1 - 4

Week of **05/04-05/08**	Day 1	Lesson 5.1.2: Optimization	Khan Academy Solving Optimization Problems	Khan Academy Solving Optimization Problems	Problems 1 - 4
	Day 2	Lesson 5.1.3: Using First and Second Derivatives	Khan Academy Using the First Derivative Test Khan Academy Using the Second Derivative Test	Khan Academy Using the First Derivative Test Khan Academy Using the Second Derivative Test	Problems 1 - 4
	Day 3	Lesson 5.1.4: Using the First and Second Derivative Tests	Khan Academy Using the First Derivative Test Khan Academy Using the Second Derivative Test	Khan Academy Using the First Derivative Test Khan Academy Using the Second Derivative Test	
	Day 4	Lesson 5.2.1: The Product Rule	Khan Academy Product Rule	Khan Academy Product Rule	
	Day 5	Lesson 5.2.2: The Chain Rule and Application (Part I)	The Chain Rule: Introduction	The Chain Rule: Introduction	Problems 1 - 4

Week of 05/11-05/15	Day 1	Lesson 5.2.3: The Chain Rule and Application (Part II)	The Chain Rule: Introduction	The Chain Rule: Introduction	
	Day 2	Lesson 5.2.4: The Quotient Rule	The Quotient Rule	The Quotient Rule	Problems 1 - 4
	Day 3	Lesson 5.2.5: More Trigonometric Derivatives	Trigonometric Derivatives	Trigonometric Derivatives	
	Day 4	Lesson 5.4.1: Chain Rule Extension of the Fundamental Theorem of Calculus	Finding the Derivative with Fundamental Theorem of Calculus: Chain Rule	Finding the Derivative with Fundamental Theorem of Calculus: Chain Rule	
	Day 5	Lesson 5.5.1: Evaluating Limits of Indeterminate Forms	Using L'Hospital's Rule	Using L'Hospital's Rule	
Week of 05/18-05/22	Day 1	Lesson 5.5.2: L'Hospital's Rule	Using L'Hospital's Rule	Using L'Hospital's Rule	
	Day 2	Chapter 5 Closure	Chapter 5 Review Problems 1 - 20		
	Day 3	Lesson 6.1.1: Exponential Functions	Derivative of Natural Base Functions e^x	Derivative of Natural Base Functions	Problems 1 - 4

	Day 4	Lesson 6.1.2: Derivatives of Exponential Functions	Derivatives of Exponential Functions	Derivatives of Exponential Functions	
	Day 5	Lesson 6.1.3: Derivatives Using Multiple Tools	Selecting a Differentiation Strategy Selecting procedures for calculating derivatives	Selecting a Differentiation Strategy Selecting procedures for calculating derivatives	
Week of 05/25-05/29	Day 1	Lesson 6.1.4: Integration of Exponential Functions	Selecting a Differentiation Strategy Selecting procedures for calculating derivatives	Selecting a Differentiation Strategy Selecting procedures for calculating derivatives	Problems 1 - 4
	Day 2	Lesson 6.2.1: Implicit Differentiation	Implicit Differentiation	Implicit Differentiation	
	Day 3	Lesson 6.2.2: Implicit Differentiation Practice	Implicit Differentiation	Implicit Differentiation	
	Day 4	Lesson 6.3.1: Inverse Trigonometric Derivatives	Inverse Trigonometric Derivatives	Inverse Trigonometric Derivatives	Problems 1 - 4

	Day 5	Lesson 6.3.2: Derivatives of Natural Logarithms	Derivative of ln(x)	Derivative of ln(x)	
Week of 06/01-06/05	Day 1	Lesson 6.3.3: Derivatives of Inverse Functions	Derivatives of Inverse Functions	Derivatives of Inverse Functions	
	Day 2	Lesson 6.4.1: Mean Value	Using the Mean Value Theorem	Using the Mean Value Theorem	Problems 1 - 4
	Day 3	Lesson 6.4.2: Mean Value Theorem	Using the Mean Value Theorem	Using the Mean Value Theorem	
	Day 4	Lesson 6.4.3: Mean Value Theorem Applications	Using the Mean Value Theorem	Using the Mean Value Theorem	
	Day 5	Chapter 6 Closure	Chapter 6 Review		
Week of 06/08-06/12	Day 1	Lesson 7.1.1: Related Rates Introduction	Related Rates - Equations	Related Rates - Equations	Problems 1 - 4
	Day 2	Lesson 7.1.2: Related Rates Applications – Pythagorean Theorem	Related Rates Application - Pythagorean Theorem	Related Rates Application - Pythagorean Theorem	

	Day 3	Lesson 7.1.3: Related Rates Applications – Similar Triangles	Related Rates Applications - Similar Triangles	Related Rates Applications - Multiple Rates	
	Day 4	Lesson 7.1.4: Related Rates Applications – Choosing the Best Formula	Solving Related Rates Problems	Solving Related Rates Problems	
	Day 5	Lesson 7.1.5: Related Rates Applications - Trigonometry	Related Rates Applications - Trigonometry	Related Rates Applications - Trigonometry	
Week of 06/15-06/19	Day 1	Lesson 7.2.1: Undoing Chain Rule	Integration Using Substitution	Integration Using Substitution	
	Day 2	Lesson 7.2.2: Integration with U-Substitution	Integration Using Substitution	Integration Using Substitution	Problems 1 - 4
	Day 3	Lesson 7.2.3: Definite Integrals with U-Substitution	Integration Using Substitution	Integration Using Substitution	
	Day 4	Lesson 7.2.4: Varied Integration Techniques	Integration Using Substitution	Integration Using Substitution	
	Day 5	U-Substitution Review	Integration Using Substitution	Integration Using Substitution	

Lesson 4.1.1 – 4.1.3

For use after lesson 4.1.3

1. Find $f'(x)$ if $f(x) = 6x^2 - \frac{5}{x} + \frac{2}{\sqrt[3]{x^2}}$.

2. Is $f(x)$ differentiable at $x = 2$? Why or why not?

$$f(x) = \begin{cases} -x^2 - 3 & \text{for } x \le 2 \\ (x-4)^2 - 3 & \text{for } x > 2 \end{cases}$$

3. Evaluate: $\int_1^5 (3x^2 - 3x + 1)dx$

4. Write each integral expression as a single integral.

a. $\int_7^3 f(x)dx + \int_3^9 f(x)dx$

b. $\int_a^c f(x)dx - \int_b^c f(x)dx \quad (a < b < c)$

Lesson 4.2.1 – 4.2.3

For use after Lesson 4.2.3

1. Find $f'(x)$ if $f(x)=\dfrac{x^5-2x^2+\frac{1}{3}x-4}{x^2}$.

2. If $\int_2^5 f(x)dx=10$, find:

   a. $\int_1^4 f(x-1)dx$

   b. $\int_0^3 \big(f(x+2)+3\big)dx$

   c. $\int_6^3 f(x-1)dx$

   d. $\int_2^2 f(x)dx$

3. If $\int_0^x g(t)dt=3x^2-2x$, find:

   a. $2\int_0^4 g(t)dt$

   b. $\int_{-2}^0 g(t)dt$

   c. $\int_{-3}^5 g(t)dt$

4. Integrate.

   a. $\int 4\sin(x-2)dx$

   b. $\int \left(\frac{3}{4}x^3-5\sqrt{x}+\pi\right)dx$

Lesson 4.3.1 – 4.3.2

For use after Lesson 4.3.2

1. Find: $f'\left(\int_6^1 f(x)dx\right)$

2. Integrate: $\int f'(x)dx$

3. If $\int_1^6 f(x)dx=50$, find:

   a. $\int_3^8 \big(f(x-2)+2\big)dx$

   b. $\int_{-1}^4 f(x+3)dx$

   c. $\int_6^1 \big(f(x)+4\big)dx$

4. Find:

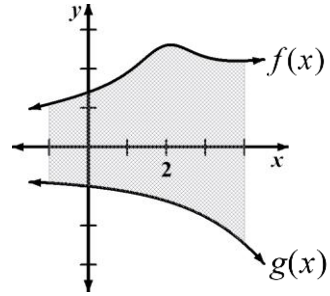
   a. $\frac{d}{dx}\int \frac{2^x}{\cos(3x-1)}dx$

   b. $\int_0^x \frac{d}{dx}(2^x\sqrt{x^2-3x+1})dx$

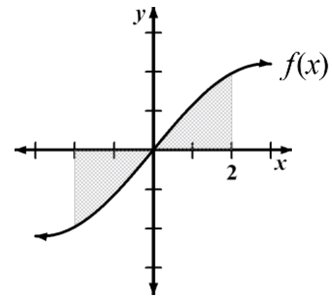
Lesson 4.4.1 – 4.4.2

For use after Lesson 4.4.2

1. Write an integral representing the shaded area shown on the graph at right from x = –1 to x = 4.



2. Write an integral representing the shaded area shown on the graph at right from x = –2 to x = 2.



3. Find the area of the region bounded by the graphs of $y=(x+2)^2-1$ and $y=-x+3$.

4. Find: a. $\int_{-1}^{x} \frac{d}{dx}(x^2-x)dx$ b. $\frac{d}{dx}\int(x\sqrt{x^2-3})dx$

Calculus Chapter 4: Review

No Calculator

1. Write a Riemann sum using 3 left endpoint rectangles under the curve $y = 2^x + x^2 + 1$ from x = 3 to x = 4. Use sigma notation. How could you modify this answer to get an exact answer?

2. Find the slope functions, $f'(x)$, for the following functions.

   a. $f(x) = 6\sqrt[4]{x^3} - \sqrt{\pi^5}$ b. $f(x) = \frac{1}{2}x^5 + \frac{1}{3}x^4 - \frac{3}{4}x$

3. Does $\int_a^b f(x)dx = \int_{a-c}^{b-c} \left(f(x-c)\right)dx$? Explain clearly why it is or is not a true statement.

4. Evaluate: $\int_{-1}^{1} \left(3z^2 - z + 1\right)dz$

5. Evaluate: $\int_6^{11} \sqrt{2}\, dx$

6. Integrate: $\int \frac{x^2 - x - 20}{x-5} dx$

7. What is the difference in area between $\int_5^{11} f(x)dx$ and $\int_5^{11} \left(f(x) + 10\right)dx$?

8. What is the Fundamental Theorem of Calculus, Part I?

9. What is the Fundamental Theorem of Calculus, Part 2?

10. A shoe falls off a rock climber at 500 feet. If the velocity of the shoe is $v(t) = -8t - 10$ feet per second, what is the velocity after 1 second? Where is the barrel after 10 seconds? Where is the barrel after *t* seconds?

11. Find: $\frac{d}{dx}\int_5^{x} \left(3x+1\right)^3 dx$

12. Find: $\int_{-1}^{x} \frac{d}{dx}(2x - x^2)dx$

13. Find: $\frac{d}{dx}\int_0^{1} (\cos(3x+1)\sqrt{x})\, dx$

For problems 14 and 15: Graph the region, show a typical rectangle, then write and evaluate the integral.

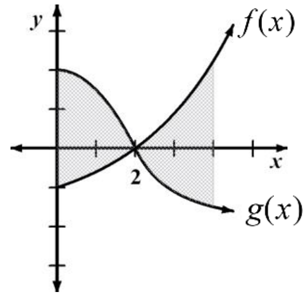
14. Find the area of the region bounded by the graphs of $y = \sqrt{x} + 2$, $x = 0$, $y = -1$, and $x = 4$.

15. Find the area of the region bounded by the graphs of $y = (x+2)^2$, $y = 4 + x$, and $y = 0$.

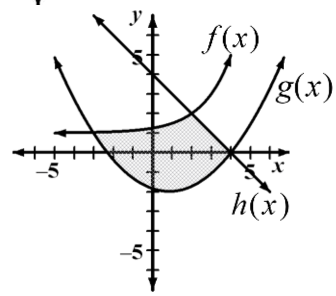
16. Find $f'(x)$ if $f(x) = 2\sqrt{\pi}$.

17. Evaluate: $\int_4^{-2} |x-2|\,dx$

18. Write an integral representing the shaded area shown on the graph at right from x = 0 to x = 4.



19. Write an integral representing the shaded area shown on the graph at right from x = –3 to x = 4.



Calculator

20. a. On your paper, sketch the function $f(x) = 5\cos x - x$. Let $x_1 = 4$, and generate a sequence of x-values to approximate the root using Newton's Method. State what happens and <u>why</u>, using your sketch and showing the progression of x-values.

    b. Explain why x_1 needs to be "reasonably" close to the root. Create your own example of a function and a poor choice of x_1 which will fail to lead to the desired root.

Lesson 5.1.1 – 5.1.2

For use after Lesson 5.1.2

1. Use Newton's Method to find the root of $f(x) = 4x^3 - 2x^2 + 3x - 1$ on the interval $[0,1]$, accurate to 3 decimal places. Let $x_1 = 0.5$.

2. A ball is thrown straight up in the air from a height of 4 feet with an initial velocity of 30 feet per second. Remember that acceleration due to gravity is -32 ft/sec^2.

    a. Find equations for $h(t)$, $v(t)$, and $a(t)$.

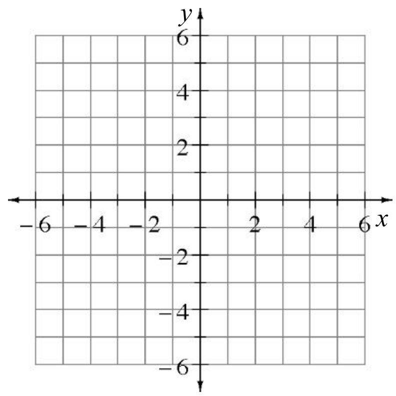
    b. What is the maximum height that the ball reaches?

    c. What is the maximum speed of the ball?

3. Find the first and second derivative of $f(x) = \frac{-2}{\sqrt[3]{x^5}} - 5\sin(x+1) + \frac{4}{7}x^9$

Lesson 5.2.1

For use after Lesson 5.2.1



1. Let $f(x) = x^3 - 3x - 2$.

   a. State the intervals on which the function is increasing and decreasing.

   b. State the intervals on which the function is concave up and concave down.

   c. Find the coordinates of all maxima, minima, and points of inflection.

   d. Sketch a graph of the function.

2. Simplify and factor: $\frac{(7y-2)^{-3}(2y+1)^{2/3}+(7y-2)^{-2}(2y+1)^{-1/3}}{(7y-2)^{-1}(2y+1)^{1/3}}$

3. If the graph of $y = \frac{px+q}{x+r}$ has a vertical asymptote at x = –4, a horizontal asymptote at y = 2, and a root at x = –5, then $p + q + r = ?$

Lesson 5.2.2 – 5.2.4

For use after Lesson 5.2.4

1. Find $\frac{dy}{dx}$.

   a. $y = (x+5)^2 + 6(x-4)(x+3)$

   b. $y = 2x\sqrt{2x+1}$

   c. $(3x^2 - 1)(2x^{-4} + x)$

   d. $y = \frac{x^3}{x^2+1}$

2. Evaluate.

   a. $\int_2^5 (x^2 - \pi x^2)dx$

   b. $\int_0^{\pi/3} (\sin x + \cos x)dx$

3. A rectangular plot of land will be bounded on one side by a river and on the other three sides by an electric fence. If 800 meters of wire are available to build the fence, what is the largest area that can be enclosed?

Lesson 5.3.1 – 5.3.2

For use after Lesson 5.3.2

1. Differentiate.

   a. $\frac{d}{dx}\left(\frac{2x^{4/3}}{\cos^2 x}\right)$

   b. $\frac{d}{df}\left(3\sqrt{f}-\sin(2f)+\frac{1}{\pi}\right)$

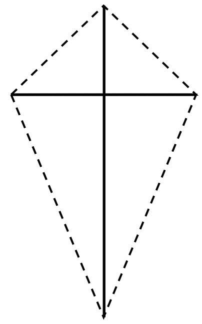
   c. $\frac{d}{dg}\left(\tan(\sin(g+1))\right)$

   d. $\frac{d}{dh}\left(h\sec h^2\right)$

2. A point moves linearly such that $a(t)=12t-14$. If the known conditions are $v(0)=8$ and $s(1)=15$, find an equation for $s(t)$.

3. Amelia is going to make a kite out of a wooden dowel that is 10 feet long. How should she cut the dowel so that she makes a kite with the largest possible area?

   Note: The wooden dowel is shown by the solid lines in the diagram at right.



Calculus Chapter 5: Review

No Calculator

1. Find $f'(x)$ for each function of a product.

   a. $f(x)=(7x+10)(2x+3)$

   b. $f(x)=(3x-2)\sqrt{x+5}$

   c. $f(x)=(5x+1)(4x^2+3x-9)$

2. Find $f'(x)$ for each function.

   a. $f(x)=(x^5+1)^2$

   b. $f(x)=\sqrt[3]{x^2-2x+1}$

   c. $f(x)=\sin^2(5x-\sqrt{x})$

3. For $y=x^3-2x^2+x-1$, find the first and second derivatives, then use them to find the maxima and minima on the interval [0, 4].

4. Use the graph of $f'(x)$ at right to determine the intervals where $f(x)$ is increasing, decreasing, concave up, and concave down.

5. Find the area between the curves $y = 2\sin x$ and $y = \frac{1}{2}x(x-\pi)$ on the interval $[0, 2\pi]$.

6. Find $f'(x)$ for each function.

   a. $f(x) = \frac{(3x^2-x)^2}{\frac{1}{4}\cos x}$ b. $f(x) = \frac{\sin x}{3(2x-1)^2}$

7. Integrate.

   a. $\int \frac{5x^5-x^3+2x^2-1}{x^5}\,dx$ b. $\int_0^1 t^2\left(\sqrt{t}-\sqrt[3]{t}\right)dt$

8. Find:

   a. $\frac{d}{dx}\int_5^x (3x+1)^3 dx$ b. $\int_{-1}^x \frac{d}{dx}(x^2-x)\,dx$

9. Sketch a graph that satisfies the following conditions:

   $f''(2) < 0$ $f'(2) = 0$ $f(2) = -1$

10. Let $f(x) = -x^3 + 3x - 2$.

    a. Find all open intervals on which $f(x)$ is increasing and decreasing.

    b. Find all open intervals on which $f(x)$ is concave up and concave down.

    c. Find the coordinates of all maximum, minimum, and point of inflection.

    d. Use the information you found in parts (a) – (c) to sketch a graph of the function.

11. Simplify.

    a. $-2 \cdot 5^{\log_5 13}$ b. $\log\left(\log_7\left(\log_3 3^7\right)\right)$

12. Find the inverse of each function.

    a. $f(x) = 4\log_3(x-1)+9$ b. $f(x) = -5\cdot 3^{x+1} - 2$

13. Find the derivative of each function.

    a. $f(x) = -7\sec(x-1)$ b. $f(x) = 6\tan x - \sqrt{x+1}$

    c. $f(x) = 2x\csc 3x$ d. $f(x) = 5(x-1)\cot^{-3} x$

14. Evaluate each limit.

    a. $\lim_{x\to 4}\frac{-\sin(\pi x)}{x-4}$ b. $\lim_{x\to 0}\frac{\sin x^2}{x^2}$

    c. $\lim_{x\to\infty}\frac{\sin x}{x}$ d. $\lim_{x\to 0}\frac{x}{\tan 3x}$

15. Write an equation to match the properties of the given function.

a. The function has a hole at x = 4, a horizontal asymptote at *y* = 2, and a vertical asymptote at x = –1.

b. The function that is graphed at right.

Calculator Okay

16. A store has been selling skateboards at the price of $40 per board, and at this price, skaters have been buying 50 boards a month. The owner of the store wishes to raise the price and estimates that for each $1 increase in price, 3 fewer boards will be sold each month. If each board costs the store $25, at what price should the store sell the boards in order to maximize profit?

17. Jane is 2 miles offshore in a boat and wishes to reach a coastal village 6 miles down a straight shoreline from the point nearest the boat. Due to the current, she can row her boat at 3 mph. She can walk at 4 mph. Where should she land her boat to reach the village in the least amount of time?

18. The height of an object thrown vertically is given by $s(t) = -16t^2 + 100t + 200$. Height is in meters and time is in seconds.

    a. What is the maximum height that the object reaches?

    b. What is the velocity of the object when *t* = 2?

    c. What is the velocity of the object when it hits the ground?

19. Aimee got a home loan of $250,000 with a 7.5% interest rate, compounded daily.

    a. How much interest will she owe at the end of the first month (30 days)?

    b. What is the annual percentage rate of her loan? (See problem 5-133 for help.)

20. Ms. Ligsay tossed a piece of chalk to Kwin, who was about to give a presentation. The velocity of the chalk with respect to the ground was $v(t) = -32t + 10$ feet/sec.

    a. When was the velocity of the chalk zero? What was happening to the chalk at that time?

    b. Find $a(t)$.

    c. If $s(0) = 5\frac{1}{3}$, find a function that represents the position of the chalk, $s(t)$.

    d. Find the time, *t*, at which Kwin caught the chalk when it was 6 feet off of the ground.

Lesson 6.1.1 – 6.1.3
For use after Lesson 6.1.3

1. Integrate.

   a. $\int (5x^{-2/3} - 2x + 5)dx$

   b. $\int x^2 \sec^2(5x^3)dx$

2. Differentiate.

   a. $\frac{d}{dx}\left(4x^{3/4} - \frac{1}{2}(\sin 3x)^{-2/3}\right)$

   b. $\frac{d}{dx}\left(\sqrt[4]{6x-1} - 7\cdot 3^x\right)$

3. Evaluate.

   a. $\lim\limits_{x\to 1}\left(\frac{x^6-1}{1-x^3}\right)$

   b. $\lim\limits_{x\to\infty}\left(\frac{x^2}{e^x}\right)$

4. A certain population of bacteria begins with 20 orgaisms. After 20 minutes there are 80 organisms.

   a. What is the percent increase?

   b. Assuming exponential growth, find an equation to model the population of this particular bacteria at any time, *t*.

   c. How fast, in oragnisms/minute, is the population of bacteria increasing when *t* = 2 hours?

Lesson 6.2.1 – 6.2.2
For use after Lesson 6.2.2

1. Integrate.

   a. $\int (\sin 2x)e^{\cos 2x}dx$ b. $\int 2^{3x}dx$

2. Find y'.

   a. $y = \sqrt[3]{(\tan 5x)^4 + \sqrt{x}}$ b. $2xy + x^2 - y^2 = y$

3. Evaluate each limit.

   a. $\lim\limits_{x \to 0} \frac{xe^x}{e^x - 1}$ b. $\lim\limits_{x \to 2} \frac{\int_4^{x^2} \sqrt{1+t}\, dt}{x-4}$

4. If $x^2 + y^2 = 25$, find the point(s) where the slope of the tangent line is $-\frac{3}{4}$.

Lesson 6.3.1 – 6.3.3
For use after Lesson 6.3.3

1. Integrate.

   a. $\int (x^2 - 3x)\sqrt{x}\, dx$ b. $\int e^x \sin\left(\frac{\pi e^x}{2}\right) dx$

2. Find $\frac{dy}{dx}$.

   a. $y = \frac{1}{2}(e^x - e^{-x})$ b. $y = \arcsin(2^x)$

   c. $y = \sqrt[3]{x}\tan(2x)$ d. $y = \csc^{-1}(x)$

3. If the perimeter of the window at right is to be 30 feet, what should the radius of the semi-circle at the top of the window be to allow the most light through?

Lesson 6.4.1 – 6.4.2
For use after Lesson 6.4.2

1. Differentiate.

   a. $y = \log_5(x^3 + 2x^2 - 3)$

   b. $y = 10^{\sec x + \csc x}$

2. Integrate.

   a. $\int \left(\sec^2(3x)\right)\left(6^{\tan(3x)}\right)dx$

   b. $\int (2x^{-5/3} - 7x^{-1} + 3)dx$

   c. $\int \frac{5}{x}\,dx$

3. If f and g are inverse functions, $f(x) = \sin(2x)$, and $g\left(\frac{1}{2}\right) = \frac{\pi}{12}$, use the formula for derivatives of inverses to find $g'\left(\frac{1}{2}\right)$.

4. Use the formula for the derivative of $\sin^{-1} x$ to find $\frac{d}{dx}\left(\frac{1}{2}\sin^{-1}\left(\frac{1}{2}\right)\right)$.

Calculus Chapter 6: Review

1. Can you guess the letter?

   a. Recall the definition of factorial: $n! = n(n-1)(n-2)\cdots(1)$. If $4! = 4\cdot3\cdot2\cdot1 = 24$, find 6! by hand. Check your answer with your calculator.

   b. Use the fact that 0! = 1, find $\sum_{k=0}^{3}\frac{1}{k!}$ without your calculator.

   c. Use your calculator to find $\sum_{k=0}^{5}\frac{1}{k!}$, $\sum_{k=0}^{10}\frac{1}{k!}$, and $\sum_{k=0}^{20}\frac{1}{k!}$.
   Record all of the decimal places that your calculator shows.

   d. Predict $\lim_{n\to\infty}\sum_{k=0}^{n}\frac{1}{k!}$.

2. Find the annual percentage rate for an account earning 6% interest:

   a. Compounded daily.

   b. Compounded 1,000 times per year.

   c. Compounded 10,000 times per year.

   d. Compounded n times per year.

   e. What is $\lim_{n\to\infty}$? Can you determine a concise way to write this number?

3. Susan and Gene are the same age. When Susan was 23, she decided to invest \$5,000 in an investment account earning 12% annual interest. Gene thought this was a good idea, so he also deposited \$5,000 in an investment account, earning 12% annual interest as well, but not until he was 29. They both decided to retire at age 65.

   a. How much money do they each have in their accounts when they retire? What is the difference?

   b. In general, how fast are their investments growing at any time, t?

   c. How fast are each of their investment accounts growing when they decide to retire? What do you notice?

   d. Approximately how long do you think it takes the accounts to double?

No Calculator

4. Differentiate.

a. $y = 5^{x^3 + \cos 2x}$

b. $f(x) = e^{3\ln(\sin x)}$

c. $f(x) = \left(7x^2 - 2x\right)^6 \left(6^{7x}\right)$

d. $y = \cos^{-1} 3x$

e. $y = \log_4(2x^3 - e^x)$

f. $y = \ln\left(\cot^2 x + \csc 2x\right)$

g. $y = \frac{1}{2(e^x - e^{-x})^2}$

h. $y = \sec(x^2)\sqrt{x}$

5. Find the following derivatives implicitly.

a. $\ln(xy) - x^3 = 3y^2$

b. $13x^3 y = 4^{xy}$

c. $3y + e^x = \sin 2y$

d. $x(x^2 + 1)(y - 1) = x - y$

6. Integrate.

a. $\int \left(\frac{2x^2 - 3x + 4}{x}\right) dx$

b. $\int (\sec 2x \tan 2x) dx$

c. $\int (\ln 5) 5^{4x} dx$

d. $\int_0^{\pi/2} \cos^3\left(\frac{1}{2}x\right) \sin\left(\frac{1}{2}x\right) dx$

7. Evaluate each limit.

a. $\lim\limits_{x \to \infty} \frac{-3x^5 - x^2 + 1}{5 + x^3 - 7x^5}$

b. $\lim\limits_{x \to 0} \frac{-3x^5 - x^2 + 1x}{5 + x^3 - 7x^5}$

c. $\lim\limits_{x \to 0} (1 + 2x)^{1/x}$

d. $\lim\limits_{x \to 0} \frac{\sin x^2}{(\sin x)^2}$

8. Find the equation of the tangent line at x = e for $y = x \ln x + 3$.

9. Suppose you are driving 90 ft/sec (about 60 mi/hr) behind a truck. When you get the opportunity to pass, you step on the accelerator, giving the car an acceleration $a = \frac{4}{\sqrt{t}}$, where a is in (feet per second) per second and t is in seconds.

a. What is the velocity function?

b. How fast are you going 9 seconds later, when you have passed the truck?

c. How far did you travel in that time?

d. What was your average velocity for the 9 second interval?

10. Find all values of x which satisfy the Mean Value Theorem for $f(x) = x^2 + 1$ on the interval [–1, 3].

11. Find the equation of the line tangent to the inverse of the given function at the given point.

a. $f(x) = x^3 + 2x - 1$; (2, 1)

b. $f(x) = x^{7/3}e^{x^2}$; (e, 1)

c. $f(x) = 5x - \cos(3x)$; (–1, 0)

d. $f(x) = x^4 + (\ln x)^3 + 2$; (3, 1)

12. Find the inverse of each function.

a. $f(x) = 3\log_5(x-1) + 2$

b. $f(x) = 9 \cdot 6^{x+7} - 4$

c. $f(x) = 3\sin(4x) - 5$

d. $f(x) = \frac{7}{2}x^{1/4} + 8$

13. Use what you know about limits and derivatives to analyze the graph of $y = \frac{3x^2}{x^2-9}$.

14. Sketch a continuous function with the following properties:

$f(1) = 1$

$f(3) = 3$

$f(-5) = 5$

$\lim_{x \to \infty} f(x) = 3$

$f'(3) = 0$

$f(x)$ is even

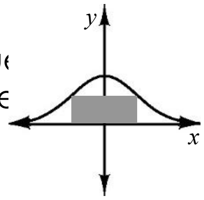
15. Evlauate the given limits.

a. $\lim\limits_{x\to 0} \frac{\int_1^{4+2x} \sin t^2 dt - \int_1^4 \sin t^2 dt}{x}$

b. $\lim\limits_{h\to 0} \frac{\int_m^{m+h} f(x)\,dx}{h}$

16. Find the equation of the line tangent to the curve $x^2 + 4xy + y^2 = 1$ at $x = 1$.

17. Given the graph of $y = 3^{-x^2}$ at right, find the x-value will yield the maximum possible area of a rectangle inscribed under the curve.



18. The velocity (in meters per second) of a particle moving in a straight line is given by $v(t) = t\sin(t^2)$. Find the total distance traveled by the partical from t = 0 to $t = \frac{\pi}{2}$ seconds.

19. Let $f(x) = \int_0^x (t+1)(t-2)(\ln t)dt$. For what values of x is $f(x)$ increasing?

20. (BC) Evaluate the improper integrals.

a. $\int_0^\infty e^{-4x}dx$

b. $\int_0^3 \frac{1}{x^3}dx$

c. $\int_0^5 \frac{1}{(x-2)^{1/3}}dx$

d. $\int_1^\infty \frac{y}{y^2+1}dy$

Lesson 7.1.1 – 7.1.3

For use after Lesson 7.1.3

1. Integrate.

   a. $\int \frac{\cot \sqrt[3]{x}}{\sqrt[3]{x}}\,dx$

   b. $\int (\tan 3x)^3 \sec^2 3x\,dx$

2. Differentiate.

   a. $f(x) = 3^{3\ln(\cos x)}$

   b. $f(x) = (7x^2 - 2x)^6(6^{7x})$

3. Find all x-values which satisfy the Mean Value Theorem on the interval [–1, 3] for $f(x) = x^2 + 1$.

4. The vase shown at right is being filled with water at a constant rate. Describe how the hieght of the water is changing with respect to time.

5. (BC) Evaluate the improper integrals.

   a. $\int_0^{2\pi} \frac{1}{x^2} \sin\left(\frac{1}{x}\right) dx$

   b. $\int_0^1 \frac{3}{\sqrt{x}}\,dx$

Lesson 7.2.1
For use after Lesson 7.2.1

1. Integrate.

   a. $\int \frac{1-\sin x}{x+\cos x} dx$

   b. $\int \frac{e^{\tan x}}{\cos^2 x} dx$

2. Differentiate.

   a. $y = \cos^{-1} 3x$

   b. $y = 5^{x^3+\cos 2x}$

3. Paul is filling up his pool at a rate of 15 cubic feet per minute. His pool is a rectangular prism with a length of 25 feet, a width of 15 feet, and a depth of 6 feet. How is the depth of the water changing with respect to time?

4. The position of a ball is given by $s(t) = t^2 - 7t + 10$ where $s(t)$ is in meters and *t* is in seconds.

   a. Find the average velocity of the ball from *t* = 0 to *t* = 5.
   b. When is the ball traveling at its average velocity?

5. Evaluate the given limits.

   a. $\lim\limits_{x\to-\infty} \frac{7x^3-5x^2-8}{-5x^3+3x^2+1}$

   b. $\lim\limits_{x\to 0} (1+4x)^{1/x}$

Lesson 7.2.3 – 7.2.4

For use after Lesson 7.2.4

1. Integrate.

   a. $\int \sin x(1+\cos^2 x)\,dx$

   b. $\int \frac{\sec^2 3x}{(\tan^3 3x)}dx$

2. Find $\frac{dy}{dx}$.

   a. $3y + e^x = \sin 2y$

   b. $y = \sqrt{x}\sec(x^2)$

3. A weather balloon is rising vertically at the rate of 5 meters per second. An observer is standing on the ground 300 meters from the point where the balloon was released. At what rate is the distance between the observer and the balloon changing when the balloon is 400 meters high?

4. Analyze the function $f(x) = \frac{2}{3}(x-1)^2(x+2)(x-5)$.