# Mathematics

# Grade 7



Dear DPSCD Families,

The Office of Mathematics is partnering with families to support Distance Learning while students are home. As your child's first teacher, we empower you to utilize the resources provided to foster a deeper understanding of grade-level mathematics.

Students in grades K-8 will work from our core curriculum, Eureka Math, utilizing this Academic Packet supported by Knowledge on the Go recorded videos. The videos have a Eureka Math instructor presenting a lesson for students to engage in grade-level mathematics. The instructor will guide students to work through the lesson by completing problems simultaneously with your child and/or asking them to pause the video for independent solving and then check. As the instructor demonstrates sample problems in the Problem Set, Application Problems, Fluency Activities, Examples and/or Exercises, parents feel free to engage your child in this work. Ask students to show work and explain their answers. When appropriate have students add models or drawings to help them solve and record answers in complete sentences.

Daily lesson guidance can be found on the pages that follow. Each day has been designed to provide you access to materials from the Eureka Math Knowledge on the Go website <a href="https://gm.greatminds.org/en-us/knowledgeonthego">https://gm.greatminds.org/en-us/knowledgeonthego</a> . After you have accessed the site, click your child's grade level, and scroll down to find the desired lesson. The resources are found at the bottom of the page and we recommend the lessons be completed in order.

Eureka Math is our core curriculum, but we also recognize it is necessary to differentiate mathematics instruction to meet all students' needs. Students took the **i-Ready** diagnostic earlier this year and it created a Learning Path for students to follow. Students work weekly on the goals set on the i-Ready Learning Path.



After their core math lesson, if able, we ask that students continue to work on their Learning Path by logging on to <u>www.clever.com</u> and selecting the i-Ready icon. In addition, students may also access the i-Ready Teacher-Assigned Lessons which would be an enrichment to grade-level content and should be utilized if extension activities are needed.

If one-on-one, live support is required, please feel free to call the **Homework Hotline** at 1-833-466-3978. Please check the <u>Homework Hotline page</u> for operating hours. We have DPSCD mathematics teachers standing by and are ready to assist.



If students need additional help, and parents have internet access, please refer to

the Homework Helper document and sign up for an account. Homework Helper provides step by step



explanations of how to work the Eureka Math problems. Also, provided on the Eureka Math Knowledge on the Go website is a plethora of **Additional Resources** that consists of Templates, Homework, Parent Tip Sheets, and more.

We appreciate your continued dedication, support and partnership with Detroit Public Schools Community District and with your assistance we can press forward with our priority: Outstanding Achievement. Be safe. Be well!

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Deputy Executive Director of K-12 Mathematics

#### Notice of Non-Discrimination

DPSCD does not discriminate on the basis of race, color, national origin, sex, sexual orientation, transgender identity, disability, age, religion, height, weight, citizenship, marital or family status, military status, ancestry, genetic information, or any other legally protected category, in its educational programs and activities, including employment and admissions Questions? Concerns? contact the Civil Rights Coordinator at (313) 240-4377 or <a href="mailto:dpscd.compliance@detroitk12.org">dpscd.compliance@detroitk12.org</a> or 3011 West Grand Boulevard, 14<sup>th</sup> Floor, Detroit MI 48202.

Parents,

Find additional resources aligned to Eureka Math here:



## ACCESSING HOMEWORK HELPER eBOOKS

STEP 1: CREATE AN ACCOUNT

Sign up for a free account at GreatMinds.org/store/signup.

#### STEP 2: ACCESS YOUR DASHBOARD

Once you have created an account at GreatMinds.org, you will be taken to your Dashboard.

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After you have logged in you can also access your Dashboard by clicking "MY DASHBOARD" in the upper right-hand corner of the site.

#### STEP 3: ENTER YOUR PRODUCT KEY

In your Dashboard you will see several buttons, select "PRODUCT KEY" and enter **H00688525** to access your Homework Helper eBook.

RECENT RESOURCES	PRODUCT KEY	REFINE	~

#### STEP 4: ACCESS YOUR HOMEWORK HELPER eBOOK

After you've entered your Product Key, select a grade-level, and the Homework Helper eBook will be added to your Dashboard. Click "LAUNCH PRODUCT" to navigate into the eBook. Note: if you are viewing the Homework Helper eBooks on a mobile device or tablet, we recommend using landscape view.

Questions? Contact us at info@GreatMinds.org.

GreatMinds.org

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# Clever—How to access DPSCD Curriculum Applications through Clever.com



1	Click on the Clever desktop shortcut or open Google Chrome and go to clever.com/in/dpscd	1	Clever	OR	Clever.com	/in/dpscd
2	Click "Log in with Active Directory" <b>Teacher's</b> will use the same credentials that they use to login to their email. <b>Student's</b> will follow the following forma listed below	2 t	Detroit Pu Not your district	blic School Distri	Clever Clever Clever Clever Clever Brigge Lag of District Admin	Login
3	Enter student's username in the space identified. The username will consist of the students ID # with @thedps.org appended on. For example 12345678@thedps.org	3	Sign in	uc schools TY DISTRICT Back	Next	
4	Enter the student's password. The password will consist of the following: First letter of first name in upper case First letter of last name in lower case 2 digit of their birth month 2 digit of their birth year O1 (male) or O2 (female) For example: Jane Doe's birthday is May 13, 200 Her password is Jd050402	4	Enter p	1234 Dassword Back	5678@thedps.org Sign in	Ř
5	Click on the application 5 you are interested in accessing	I-Ready CCP		myON ①	Office 365 Microsoft Office 365	Pearson EasyBridge Plus Pearson Easy Bridge typing.com

# **Grade 7 Mathematics**

WEEKLY DISTANCE LEARNING STUDENT SCHEDULE

# 4/14/20 to 4/17/20 Week 1 (4 days)

Directions: Target Standard(s)	<ul> <li>Parents: Assist students with accessing the "Knowledge on the Go" videos, Problem Sets in this packet, and i-Ready through the Clever app. Also, monitor student's progress while working on the videos and/or online lessons.</li> <li>Students: Click or watch the "Knowledge on the Go" video each day and complete the daily Problem Set. Visit i-ready to continue your learning path and complete Teacher-Assigned lessons.</li> <li>6.RP.A.3c ,7.RP.A.2.c, 7.RP.A.3</li> </ul>
Module Topic	Module 4: Percent and Proportional Relationships Topic A: Finding the Whole
Materials Needed :	<ul> <li>Access to Knowledge on the Go Lesson Videos &amp; Resources including Templates &amp; Homework Helpers which provide guidance with worked examples for each lesson.</li> <li>Clever Access for i-Ready (see links and QR codes below)</li> <li>Paper, Pencil, Academic Packet including Problem Sets .</li> </ul>

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	Knowledge on the Go Vic	<u>leos clever.com Ac</u>	dditional Resources
	Daily Lesson	Extension	Intervention
	(50 minutes)	(10-15 minutes)	(15 minutes)
Day 1	Knowledge on the Go Lesson Materials	i-Ready	i-Ready
-	for Module 4, Lesson 1	"Teacher Assigned"	"My Path"
		Lesson	Lesson
		<u>clever.com</u>	<u>clever.com</u>
Day 2	Knowledge on the Go Lesson Materials	i-Ready	i-Ready
-	for Module 4, Lesson 2	"Teacher Assigned"	"My Path"
		Lesson	Lesson
Day 3	Knowledge on the Go Lesson Materials	i-Ready	i-Ready
-	for Module 4, Lesson 3	"Teacher Assigned"	"My Path"
		Lesson	Lesson
Day 4	Knowledge on the Go Lesson Materials	i-Ready	i-Ready
-	for Module 4, Lesson 4	"Teacher Assigned"	"My Path"
		Lesson	Lesson

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

	Lesson 1
Standard	6.RP.A.3c
Learning Target	Students understand that P percent is the number P/100 and that the symbol % means percent. Students convert between a fraction, decimal, and percent, including percents that are less than 1% or greater than 100%. Students write a non-whole number percent as a complex fraction.
Launch	Recommended: : Students will view the "Knowledge on the Go" video for Module 4, Lesson 1. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 4</b> , <b>Lesson 1</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 4 Lesson 1.
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

	Lesson 2
Standard	7.RP.A.2.c
Learning Target	Students understand that the whole is 100% and use the formula Part = Percent × Whole to problem-solve when given two terms out of three from the part, whole, and percent. Students solve word problems involving percent using expressions, equations, and numeric and visual models.
Launch	Recommended: : Students will view the "Knowledge on the Go" video for Module 4, Lesson 2. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 4</b> , <b>Lesson 2</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 4 Lesson 2.
Extend	Recommended: Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.
Intervention	<b>Recommended:</b> Students will work on their individual Learning Path (My
	Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

Standard	7.RP.A.2.c, 7.RP.A.3
Learning Target	Students use the context of a word problem to determine which of two quantities represents the whole. Students understand that the whole is 100% and think of one quantity as a percent of another using the formula Quantity=Percent × Whole to problem-solve when given two terms out of three from a quantity, whole, and percent. When comparing two quantities, students compute percent more or percent less using algebraic, numeric, and visual models.
Launch	Recommended: : Students will view the "Knowledge on the Go" video for Module 4, Lesson 3. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 4</b> , <b>Lesson 3</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 4 Lesson 3.
Extend	Recommended: Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready. SCAN ME
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.

Lesson 3

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

	Lesson 4
Standard	7.RP.A.2.c, 7.RP.A.3
Learning	Students solve percent problems when one quantity is a certain percent
Target	more or less than another.
	Students solve percent problems involving a percent increase or decrease.
Launch	Recommended: : Students will view the "Knowledge on the Go" video for Module 4, Lesson 4. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 4</b> , <b>Lesson 4</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 4 Lesson 4.
Extend	Recommended: Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.

# **Grade 7 Mathematics**

WEEKLY DISTANCE LEARNING STUDENT SCHEDULE

# 4/20/20 to 4/24/20 Week 2 (5 days)

Directions:	<b>Parents:</b> Assist students with accessing the "Knowledge on the Go" videos, Problem Sets in this packet, and i-Ready through the Clever app. Also, monitor student's progress while working on the videos and/or online lessons. <b>Students:</b> Click or watch the "Knowledge on the Go" video each day and complete the daily Problem Set. Visit i-ready to continue your learning path and complete Teacher-Assigned lessons.
Target Standard(s)	7.RP.A.2.c, 7.RP.A.3, 7.EE.B.3
Module Topic	Module 4: Percent and Proportional Relationships Topic A: Finding the Whole Topic B: Percent Problems Including More than One Whole
Materials Needed:	<ul> <li>Access to Knowledge on the Go Lesson Videos &amp; Resources including Templates &amp; Homework Helpers which provide guidance with worked examples for each lesson.</li> </ul>

- Clever Access for i-Ready (see links and QR codes below)
- Paper, Pencil, Academic Packet including Problem Sets

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Knowledge on the Go Videos clever.com

Additional Resources

	Daily Lesson	Extension	Intervention
	(50 minutes)	(10-15 minutes)	(15 minutes)
Day 5	Knowledge on the Go Lesson Materials for Module 4,	i-Ready	i-Ready
	Lesson 5	"Teacher Assigned" Lesson	"My Path"
		<u>clever.com</u>	Lesson
			<u>clever.com</u>
Day 6	Knowledge on the Go Lesson Materials for Module 4,	i-Ready	i-Ready
	Lesson 6	"Teacher Assigned" Lesson	"My Path"
			Lesson
Day 7	Knowledge on the Go Lesson Materials for Module 4,	i-Ready	i-Ready
	Lesson 7	"Teacher Assigned" Lesson	"My Path"
			Lesson
Day 8	Knowledge on the Go Lesson Materials for Module 4,	i-Ready	i-Ready
	Lesson 8	"Teacher Assigned" Lesson	"My Path"
			Lesson
Day 9	Knowledge on the Go Lesson Materials for Module 4,	i-Ready	i-Ready
	Lesson 9	"Teacher Assigned" Lesson	"My Path"
			Lesson

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

Lesson 5		
Standard	7.RP.A.3	
Learning Target	Students find 100% of a quantity (the whole) when given a quantity that is a percent of the whole by using a variety of methods including finding 1%, equations, mental math using factors of 100, and double number line models. Students solve word problems involving finding 100% of a given quantity with and without using equations.	
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 4</b> , <b>Lesson 5</b> . Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.	
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 4</b> , <b>Lesson 5</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.	
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 4 Lesson 5.	
Extend	Recommended: Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.	
Intervention	<b>Recommended</b> : Students will work on their individual Learning Path (My Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.	

### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

	Lesson 6	
Standard	7.RP.A.2.c, 7.RP.A.3	
Learning Target	Students solve various types of percent problems by identifying the type of percent problem and applying appropriate strategies. Students extend mental math practices to mentally calculate the part, the percent, or the whole in percent word problems.	
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 4</b> , Lesson 6. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.	
Guided Practice	<b>Recommended:</b> Students will complete the Problem Set for <b>Module 4</b> , <b>Lesson 6</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.	
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 4 Lesson 6.	
Extend	Recommended: Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready. SCAN ME	
Intervention	<b>Recommended</b> : Students will work on their individual Learning Path (My Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.	

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

	Lesson 7		
Standard	7.RP.A.2.c, 7.RP.A.3, 7.EE.B.3		
Learning	Students understand the terms original price, selling price, markup,		
Target	markdown, markup rate, and markdown rate.		
	Students identify the original price as the whole and use their knowledge of		
	percent and proportional relationships to solve multi-step markup and		
	markdown problems.		
	Students understand equations for markup and markdown problems and		
	Use them to solve for Unknown quantities in such scenarios.		
Launch	<b>Recommended:</b> Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 4</b> , <b>Lesson 7</b> . Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.		
	SCAN ME		
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 4</b> , <b>Lesson 7</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.		
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 4 Lesson 7.		
Extend	Recommended: Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.		
Intervention	<b>Recommended</b> : Students will work on their individual Learning Path (My Path) in i-Ready. Visit Clever.com to access i-Ready.		

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

Lesson 8

Standard	7.EE.B.3, 7.RP.A.3	
Learning Target	Given the exact value, x, of a quantity and an approximate value, a, of the quantity, students use the absolute error, $ a-x $ , to compute the percent error by using the formula $ a-x / x  \times 100\%$ . Students understand the meaning of percent error as the percent the absolute error is of the exact value. Students understand that when an exact value is not known, an estimate of the percent error can still be computed when given a range determined by two inclusive values (e.g., if there are known to be between 6,000 and 7,000 black bears in New York, but the exact number is not known, the percent error can be estimated to be (1,000/6,000)(100%) at most, which is 16.2(3%)	
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 4</b> , Lesson 8. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.	
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 4</b> , <b>Lesson 8</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.	
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 4 Lesson 8.	
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.	
Intervention	<b>Recommended</b> : Students will work on their individual Learning Path (My Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.	

## **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

	Lesson 9	
Standard	7.RP.A.2.c, 7.RP.A.3, 7.EE.B.3	
Learning Target	Students solve percent problems where quantities and percents change. Students use a variety of methods to solve problems where quantities and percents change, including double number lines, visual models, and equations.	
Launch	Recommended: : Students will view the "Knowledge on the Go" video for Module 4, Lesson 9. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.	
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 4</b> , <b>Lesson 9</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.	
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 4 Lesson 9.	
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.	
Intervention	<b>Recommended</b> : Students will work on their individual Learning Path (My Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.	

# **Grade 7 Mathematics**

WEEKLY DISTANCE LEARNING STUDENT SCHEDULE

# 4/27/20 to 5/01/20 Week 3 (5 days)

Directions:	<ul> <li>Parents: Assist students with accessing the "Knowledge on the Go" videos, Problem Sets in this packet, and i-Ready through the Clever app. Also, monitor student's progress while working on the videos and/or online lessons.</li> <li>Students: Click or watch the "Knowledge on the Go" video each day and complete the daily Problem Set. Visit i-ready to continue your learning path</li> </ul>
	and complete Teacher-Assigned lessons.
Target Standard(s)	7.RP.A.2.b, 7.RP.A.2.c, 7.RP.A.3, 7.EE.B.4.a, 7.G.A.1
Module	Module 4: Percent and Proportional Relationships
Торіс	Topic B: Percent Problems Including More than One Whole Topic C: Scale Drawings
Materials Needed:	<ul> <li>Access to Knowledge on the Go Lesson Videos &amp; Resources including Templates &amp; Homework Helpers which provide guidance with worked examples for each lesson.</li> </ul>

- Clever Access for i-Ready (see links and QR codes below)
- Paper, Pencil, Academic Packet including Problem Sets

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Knowledge on the Go Videos clever.com

Additional Resources

	Daily Lesson	Extension	Intervention
	(50 minutes)	(10-15 minutes)	(15 minutes)
Day 10	Knowledge on the Go Lesson Materials for Module 4,	i-Ready	i-Ready
	Lesson 10	"Teacher Assigned" Lesson	"My Path"
		<u>clever.com</u>	Lesson
			<u>clever.com</u>
Day 11	Knowledge on the Go Lesson Materials for Module 4,	i-Ready	i-Ready
	Lesson 11	"Teacher Assigned" Lesson	"My Path"
			Lesson
Day 12	Knowledge on the Go Lesson Materials for Module 4,	i-Ready	i-Ready
	Lesson 12	"Teacher Assigned" Lesson	"My Path"
			Lesson
Day 13	Knowledge on the Go Lesson Materials for Module 4,	i-Ready	i-Ready
	Lesson 13	"Teacher Assigned" Lesson	"My Path"
			Lesson
Day 14	Knowledge on the Go Lesson Materials for Module 4,	i-Ready	i-Ready
	Lesson 14	"Teacher Assigned" Lesson	"My Path"
			Lesson

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

Lesson 10		
Standard	7.EE.B.4.a, 7.RP.A.2.c, 7.RP.A.3	
Learning Target	Students solve simple interest problems using the formula I = Prt, where I represents interest, P represents principal, r represents interest rate, and t represents time. When using the formula I = Prt, students recognize that units for both interest rate and time must be compatible; students convert the units when necessary.	
Launch	Recommended: : Students will view the "Knowledge on the Go" video for Module 4, Lesson 10. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.	
Guided Practice	<b>Recommended:</b> Students will complete the Problem Set for <b>Module 4</b> , <b>Lesson 10</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.	
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 4 Lesson 10.	
Extend	Recommended: Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.	
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.	

### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

Standard	7.EE.B.4.a, 7.RP.A.2, 7.RP.A.3	
Learning	Students solve percent problems where quantities and percents change.	
Target	Students use a variety of methods to solve problems where quantities and percents change, including double number lines, visual models, and	
	equations.	
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 4</b> , Lesson 11. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.	
Guided	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 4</b> ,	
Practice	<b>Lesson 11</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.	
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 4 Lesson 11.	
Extend	Recommended: Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.	
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.	

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

Lesson 12

Standard	7.G.A.1, 7.RP.A.2.b	
Learning Target	Given a scale factor as a percent, students make a scale drawing of a picture or geometric figure using that scale, recognizing that the enlarged or reduced distances in a scale drawing are proportional to the corresponding distances in the original picture. Students understand scale factor to be the constant of proportionality. Students make scale drawings in which the horizontal and vertical scales are different.	
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 4</b> , Lesson 12. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos. SCAN ME	
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 4</b> , <b>Lesson 12</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.	
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 4 Lesson 12.	
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready. SCAN ME	
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.	

### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

	Lesson 13	
Standard	7.G.A.1	
Learning Target	Given Drawing 1 and Drawing 2 (a scale model of Drawing 1 with scale factor), students understand that Drawing 1 is also a scale model of Drawing 2 and compute the scale factor. Given three drawings that are scale drawings of each other and two scale factors, students compute the other related scale factor.	
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 4</b> , Lesson 13. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.	
Guided	<b>Recommended: :</b> Students will complete the Problem Set for Module 4,	
Practice	<b>Lesson 13</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.	
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 4 Lesson 13.	
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.	
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.	

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

	Lesson 14	
Standard	7.G.A.1	
Learning	Given a scale drawing, students compute the lengths in the actual picture	
Target	using the scale factor.	
Launch	Recommended: : Students will view the "Knowledge on the Go" video for Module 4, Lesson 14. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.	
Guided	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 4</b> ,	
Practice	<b>Lesson 14</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.	
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 4 Lesson 14.	
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.	
Intervention	Recommended: Students will work on their individual Learning Path (My	
Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.		

# **Grade 7 Mathematics**

WEEKLY DISTANCE LEARNING STUDENT SCHEDULE

	5/04/20 to 5/08/20 V	Week 4 (5 days)
Direction	ns: <u>Parents:</u> Assist students with ac Problem Sets in this packet, ar monitor student's progress wh lessons. <u>Students:</u> Click or watch the " complete the daily Problem S and complete Teacher-Assign	ccessing the "Knowledge on the Go" videos, nd i-Ready through the Clever app. Also, nile working on the videos and/or online "Knowledge on the Go" video each day and Set. Visit i-ready to continue your learning path ned lessons.
Target S	andard(s) 7.G.A.1, 7.EE.B.3, 7.E.B.4.a, 7.R	P.A.3, 7.SP.C.5
Module Topic Material	Module 4: Percent and Propo Topic C: Scale Drawings Topic D: Population, Mixture, c Module 5: Statistics and Probo Topic A: Calculating and Inter • Access to Knowledge	ortional Relationships and Counting Problems Involving Percents ability rpreting Probabilities ae on the Go Lesson Videos & Resources
	including Templates guidance with work Clever Access for i-l Paper, Pencil, Acac SCAN ME Knowledge on the Go Videos	A Homework Helpers which provide ked examples for each lesson. Ready (see links and QR codes below) demic Packet including Problem Sets SCAN ME SCAN ME SCAN ME SCAN ME SCAN ME Module 4 Module 5
	Daily Lesson	Extension Intervention
	(50 minutes)	(10-15 minutes) (15 minutes)
Day 15	Knowledge on the Go Lesson Materials for Module 4, Less <u>15</u>	son     i-Ready     i-Ready       "Teacher Assigned" Lesson     "My Path"       clever.com     Lesson       clever.com     clever.com
Day 16	Knowledge on the Go Lesson Materials for Module 4, Less <u>16</u>	son     i-Ready     i-Ready       "Teacher Assigned" Lesson     "My Path"       Lesson
Day 17	Knowledge on the Go Lesson Materials for Module 4, Less <u>17</u>	soni-Readyi-Ready"Teacher Assigned" Lesson"My Path"Lesson
Day 18	Knowledge on the Go Lesson Materials for Module 4, Less <u>18</u>	son i-Ready i-Ready "Teacher Assigned" Lesson "My Path" Lesson
Day 19	Knowledge on the Go Lesson Materials for Module 5, Less	son i-Ready i-Ready
	1	"Teacher Assigned" Lesson "My Path" Lesson

## **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

	Lesson 15	
Standard	7.G.A.1	
Learning	Students solve area problems related to scale drawings and percent by	
Target	using the fact that an area, $A'$ , of a scale drawing is $k^2$ times the	
	corresponding area, A , in the original drawing, where k is the scale factor.	
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 4</b> , Lesson 15. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.	
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 4</b> , <b>Lesson 15</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.	
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 4 Lesson 15.	
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.	
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.	

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

	Lesson 10	
Standard	7.EE.B.3, 7.RP.A.3	
Learning	Students write and use algebraic expressions and equations to solve	
Target	percent word problems related to populations of people and compilations.	
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 4</b> , Lesson 16. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.	
Guided	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 4</b> ,	
Practice	<b>Lesson 16</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.	
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 4 Lesson 16.	
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.	
Intervention	Recommended: : Students will work on their individual Learning Path (My	
	Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.	
Click the Knowledge on the Collesson Materials link or scan the Knowledge on the Co.O.P. Code in the Materials so		

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

	Lesson 17	
Standard	7.EE.B.3, 7.EE.B.4.a, 7.RP.A.3	
Learning	Students write and use algebraic expressions and equations to solve	
Target	percent word problems related to mixtures.	
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 4</b> , Lesson 17. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.	
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 4</b> , <b>Lesson 17</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.	
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 4 Lesson 17.	
Extend	Recommended: Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.	
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My	
	Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.	

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

	Lesson 18	
Standard	7.EE.B.3, 7.RP.A.3	
Learning	Students solve counting problems related to computing percents.	
Target		
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 4</b> , Lesson 18. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.	
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 4</b> , <b>Lesson 18</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.	
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 4 Lesson 18.	
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready. SCAN ME	
Intervention	Recommended: : Students will work on their individual Learning Path (My	
	Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.	

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

	Lesson 19	
Standard	7.SP.C.5	
Learning Target	Students understand that a probability is a number between 0 and 1 that represents the likelihood that an event will occur. Students interpret a probability as the proportion of the time that an event occurs when a chance experiment is repeated many times.	
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 5</b> , Lesson 1. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.	
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 5</b> , <b>Lesson 1</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.	
Closing	<b>Recommended</b> : Students will reflect and share their learning on Module 5 Lesson 1.	
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready. SCAN ME	
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.	

# **Grade 6 Mathematics**

WEEKLY DISTANCE LEARNING STUDENT SCHEDULE

5	5/11/20 to 5/15/20 Week 5 (5 days)	
Directions:	<b>Parents:</b> Assist students with accessing the "Knowledge on the Go" videos, Problem Sets in this packet, and i-Ready through the Clever app. Also, monitor student's progress while working on the videos and/or online lessons.	
Target Standard(s)	<b>Students:</b> Click or watch the "Knowledge on the Go" video each day and complete the daily Problem Set. Visit i-ready to continue your learning path and complete Teacher-Assigned lessons. 7.SP.C.5, 7.SP.C.6, 7.SP.C.7.a, 7.SP.C.7.b, 7.SP.C.8a, 7.SP.C.8b	
Module Topic Materials Needed:	<ul> <li>Module 5: Statistics and Probability</li> <li>Topic A: Calculating and Interpreting Probabilities</li> <li>Access to Knowledge on the Go Lesson Videos &amp; Resources including Templates &amp; Homework Helpers which provide guidance with worked examples for each lesson.</li> <li>Clever Access for i-Ready (see links and QR codes below)</li> <li>Paper, Pencil, Academic Packet including Problem Sets</li> </ul>	





Knowledge on the Go Videos clever.com

Additional Resources

	Daily Lesson	Extension	Intervention
	(50 minutes)	(10-15 minutes)	(15 minutes)
Day 20	Knowledge on the Go Lesson Materials for Module	i-Ready	i-Ready
	<u>5, Lesson 2</u>	"Teacher Assigned"	"My Path"
		Lesson	Lesson
		<u>clever.com</u>	<u>clever.com</u>
Day 21	Knowledge on the Go Lesson Materials for Module	i-Ready	i-Ready
	<u>5, Lesson 3</u>	"Teacher Assigned"	"My Path"
		Lesson	Lesson
Day 22	Knowledge on the Go Lesson Materials for Module	i-Ready	i-Ready
	<u>5, Lesson 4</u>	"Teacher Assigned"	"My Path"
		Lesson	Lesson
Day 23	Knowledge on the Go Lesson Materials for Module	i-Ready	i-Ready
	<u>5, Lesson 5</u>	"Teacher Assigned"	"My Path"
		Lesson	Lesson
Day 24	Knowledge on the Go Lesson Materials for Module	i-Ready	i-Ready
	<u>5, Lesson 6</u>	"Teacher Assigned"	"My Path"
		Lesson	Lesson

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

	Lesson 20	
Standard	7.SP.C.6	
Learning	Students estimate probabilities by collecting data on an outcome of a	
Target	chance experiment.	
	Students use given data to estimate probabilities.	
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 5</b> , Lesson 2. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.	
Guided	Recommended: : Students will complete the Problem Set for Module 5,	
Practice	<b>Lesson 2</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.	
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 5 Lesson 2.	
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready. SCAN ME	
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.	

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

Lesson 21

Standard	7.SP.C.6	
Learning Target	Students determine the possible outcomes for simple chance experiments. Given a description of a simple chance experiment, students determine the sample space for the experiment. Given a description of a chance experiment and an event, students determine for which outcomes in the sample space the event will occur. Students distinguish between chance experiments with equally likely outcomes and chance experiments for which the outcomes are not equally likely.	
Launch	Recommended: : Students will view the "Knowledge on the Go" video for Module 5, Lesson 3. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.	
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 5</b> , <b>Lesson 3</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.	
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 5 Lesson 3.	
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.	
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.	

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

	Lesson 22	
Standard	7.SP.C.6, 7.SP.C.7.a	
Learning	Students calculate probabilities of events for chance experiments that	
Target	have equally likely outcomes.	
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> Go" video for <b>Module 5</b> , Lesson 4. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.	
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 5</b> , <b>Lesson 4</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.	
Closing	<b>Recommended</b> : Students will reflect and share their learning on Module 5 Lesson 4.	
Extend	Recommended: Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready. SCAN ME	
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My	

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

	Lesson 23		
Standard	7.SP.C.6, 7.SP.C.7.b		
Learning	Students calculate probabilities for chance experiments that do not have		
Target	equally likely outcomes.		
Launch	Recommended: : Students will view the "Knowledge on the Go" video for Module 5, Lesson 5. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.		
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 5</b> , <b>Lesson 5</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.		
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 5 Lesson 5.		
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready. SCAN ME		
Intervention	Recommended: : Students will work on their individual Learning Path (My		
	Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.		

### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

	Lesson 24	
Standard	7.SP.C.8.a, 7.SP.C.8.b	
Learning Target	Given a description of a chance experiment that can be thought of as	
larger	organize and represent the outcomes in the sample space.	
	Students calculate probabilities of compound events.	
Launch *Parents will assist students with accessing the "Knowledge on the Go" videos or Clever App Guided	Recommended: : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 5</b> , Lesson 6. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos. SCAN ME Recommended: : Students will complete the Problem Set for <b>Module 5</b> ,	
Practice	<b>Lesson 6</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.	
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 5 Lesson 6.	
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready. SCAN ME	
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My	
	ruinj in Freduy. Visil <u>Cievel.com</u> 10 access Freduy.	

# **Grade 7 Mathematics**

WEEKLY DISTANCE LEARNING STUDENT SCHEDULE

5,	/18/20 to 5/22/20 Week 6 (5 days)
Directions:	<b>Parents:</b> Assist students with accessing the "Knowledge on the Go" videos, Problem Sets in this packet, and i-Ready through the Clever app. Also, monitor student's progress while working on the videos and/or online lessons.
	<u>Students:</u> Click or watch the "Knowledge on the Go" video each day and complete the daily Problem Set. Visit i-ready to continue your learning path and complete Teacher-Assigned lessons.
Target Standard(s)	7.SP.C.6, 7.SP.C.7, 7.SP.C.8a, 7.SP.C.8b, 7.SP.C.8.c
Module	Module 5: Statistics and Probability
Торіс	Topic A: Calculating and Interpreting Probabilities Topic B: Estimating Probabilities
Materials Needed:	<ul> <li>Access to Knowledge on the Go Lesson Videos &amp; Resources including Templates &amp; Homework Helpers which provide guidance with worked examples for each lesson.</li> <li>Clever Access for i-Ready (see links and QR codes below)</li> <li>Paper, Pencil, Academic Packet including Problem Sets</li> </ul>







Knowledge on the Go Videos clever.com

Additional Resources

	Daily Lesson	Extension	Intervention
	(50 minutes)	(10-15 minutes)	(15 minutes)
Day 25	Knowledge on the Go Lesson Materials for Module 5,	i-Ready	i-Ready
	Lesson 7	"Teacher Assigned" Lesson	"My Path"
		<u>clever.com</u>	Lesson
			<u>clever.com</u>
Day 26	Knowledge on the Go Lesson Materials for Module 5,	i-Ready	i-Ready
	Lesson 8	"Teacher Assigned" Lesson	"My Path"
			Lesson
Day 27	Knowledge on the Go Lesson Materials for Module 5,	i-Ready	i-Ready
	Lesson 9	"Teacher Assigned" Lesson	"My Path"
			Lesson
Day 28	Knowledge on the Go Lesson Materials for Module 5,	i-Ready	i-Ready
	Lesson 10	"Teacher Assigned" Lesson	"My Path"
			Lesson
Day 29	Knowledge on the Go Lesson Materials for Module 5,	i-Ready	i-Ready
	Lesson 11	"Teacher Assigned" Lesson	"My Path"
			Lesson

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

	Lesson 25	
Standard	7.SP.C.8.a, 7.SP.C.8.b	
Learning	Students calculate probabilities of compound events.	
Target		
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 5</b> , Lesson 7. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.	
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 5</b> , <b>Lesson 7</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.	
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 5 Lesson 7.	
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready. SCAN ME	
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My Path) in i-Ready. Visit Clever.com to access i-Ready.	
#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

Lesson 26

Standard	7.SP.C.6, 7.SP.C.7			
Learning Target	Given theoretical probabilities based on a chance experiment, students describe what they expect to see when they observe many outcomes of the experiment. Students distinguish between theoretical probabilities and estimated probabilities. Students understand that probabilities can be estimated based on observing outcomes of a chance experiment.			
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 5</b> , Lesson 8. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.			
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 5</b> , <b>Lesson 8</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.			
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 5 Lesson 8.			
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready. SCAN ME			
Intervention	<b>Recommended:</b> Students will work on their individual Learning Path (My			
	Tany in Freday. Visit <u>Clevel.com</u> to access Freday.			

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

Lesson 27				
Standard	7.SP.C.7			
Learning	Students compare estimated probabilities to those predicted by a			
Target	probability model.			
Launch	<b>Recommended:</b> : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 5</b> , <b>Lesson 9</b> . Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.			
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 5</b> , <b>Lesson 9</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.			
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 5 Lesson 9.			
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready. SCAN ME			
Intervention	Recommended: : Students will work on their individual Learning Path (My			
	Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.			

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

Lesson 28			
Standard	7.SP.C.8.a, 7.SP.C.8.c		
Learning Target	<ul> <li>Students learn simulation as a method for estimating probabilities that can be used for problems in which it is difficult to collect data by experimentation or by developing theoretical probability models.</li> <li>Students learn how to perform simulations to estimate probabilities.</li> <li>Students use various devices to perform simulations (e.g., coin, number cube, cards).</li> <li>Students compare estimated probabilities from simulations to theoretical probabilities.</li> </ul>		
Launch Guided	Recommended: : Students will view the "Knowledge on the         Go" video for Module 5, Lesson 10. Scan the Knowledge on         the Go QR Code or click the link to access the video. We         encourage parents to assist students with accessing and         engaging with the "Knowledge on the Go" videos.         SCAN ME         Recommended: : Students will complete the Problem Set for Module 5,		
Practice	<b>Lesson 10</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.		
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 5 Lesson 10.		
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready. SCAN ME		
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.		

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

Lesson 29			
Standard	7.SP.C.8.a, 7.SP.C.8.c		
Learning	Students design their own simulations.		
Target	Students learn to use two more devices in simulations: colored disks and a		
	random number table.		
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 5</b> , Lesson 11. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.		
Guided	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 5</b> ,		
Practice	<b>Lesson 11</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.		
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 5 Lesson 11.		
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready. SCAN ME		
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My		
	Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.		
Click the Knowledge on the Gollesson Materials link or scan the Knowledge on the Go OP Code in the Materials sec			

# **Grade 7 Mathematics**

WEEKLY DISTANCE LEARNING STUDENT SCHEDULE

	5/26/20 to 5/29/20 Week 7 (4 days)				
Direction Target St	<ul> <li>ctions: Parents: Assist students with accessing the "Knowledge on the Go" videos Problem Sets in this packet, and i-Ready through the Clever app. Also, monitor student's progress while working on the videos and/or online lessons.</li> <li>Students: Click or watch the "Knowledge on the Go" video each day and complete the daily Problem Set. Visit i-ready to continue your learning por and complete Teacher-Assigned lessons.</li> <li>et Standard(s) 7.SP.C.6, 7.SP.C.7.b, 7.SP.A.1, 7.SP.A.2</li> </ul>			n the Go" videos, rer app. Also, nd/or online eo each day and your learning path	
Module Topic	ModuleModule 5: Statistics and ProbabilityTopicTopic B: Estimating ProbabilitiesTopic C: Random Sampling and Estimating Population Characteristic		naracteristics		
Materials Needed:		<ul> <li>Access to Knowledge on a including Templates &amp; Horwith worked examples for</li> <li>Clever Access for i-Ready</li> <li>Paper, Pencil, Academic</li> </ul>	the Go Lesson Videos & F mework Helpers which pr each lesson. (see links and QR codes Packet including Problem SCAN ME <u>clever.com</u> <u>Additio</u>	Resources ovide guidance below) n Sets SCAN ME SCAN ME	
		Daily Lesson	Extension		
		(50 minutes)	(10-15 minutes)	(15 minutes)	
Day 30	Knowledge o	n the Go Lesson Materials for Module	I-Ready	i-Ready	
		<u>5, Lesson 12</u>	l esson	"My Path" Lesson	
Day 31	Knowledge o	n the Go Lesson Materials for Module	i-Ready	i-Ready	
24,01		5 Lesson 13	"Teacher Assigned"	"My Path"	

Day 31	knowledge on the Go Lesson Materials for Module	I-Reddy	I-Ready
	<u>5, Lesson 13</u>	"Teacher Assigned"	"My Path"
		Lesson	Lesson
Day 32	Knowledge on the Go Lesson Materials for Module	i-Ready	i-Ready
	<u>5, Lesson 14</u>	"Teacher Assigned"	"My Path"
		Lesson	Lesson
Day 33	Knowledge on the Go Lesson Materials for Module	i-Ready	i-Ready
	<u>5, Lesson 15</u>	"Teacher Assigned"	"My Path"
		Lesson	Lesson

Click the Knowledge on the Go Lesson Materials Link or scan the QR Reader Code in the materials section. Then, scroll down and click on corresponding Module and Lesson.

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

	Lesson 30			
Standard	7.SP.C.6, 7.SP.C.7.b			
Learning	Students use estimated probabilities to judge whether a given probability			
Target	model is plausible.			
	Students use estimated probabilities to make informed decisions.			
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 5</b> , Lesson 12. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.			
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 5</b> , <b>Lesson 12</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.			
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 5 Lesson 12.			
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.			
Intervention	Recommended: : Students will work on their individual Learning Path (My			
	Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.			

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

Lesson 31			
Standard	7.SP.A.1		
Learning Target	Students differentiate between a population and a sample. Students differentiate between a population characteristic and a sample statistic. Students investigate statistical questions that involve generalizing from a sample to a larger population.		
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 5</b> , Lesson 13. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.		
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 5</b> , <b>Lesson 13</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.		
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 5 Lesson 13.		
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.		
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.		

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

Standard	7.SP.A.1, 7.SP.A.2		
Learning	Students understand that how a sample is selected is important if the goal is		
Target	to generalize from the sample to a larger population.		
_	Students understand that random selection from a population tends to		
	produce samples that are representative of the population.		
Launch	<b>Recommended:</b> : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 5</b> , Lesson 14. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.		
Guided	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 5</b> .		
Practice	Lesson 14 from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.		
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 5 Lesson 14.		
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.		
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.		

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

Lesson 33				
Standard	7.SP.A.1, 7.SP.A.2			
Learning	Students select a random sample from a population.			
Target	Students begin to develop an understanding of sampling variability.			
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 5</b> , Lesson 15. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.			
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 5</b> , <b>Lesson 15</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.			
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 5 Lesson 15.			
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.			
Intervention	Recommended: : Students will work on their individual Learning Path (My			
	Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.			

# **Grade 7 Mathematics**

WEEKLY DISTANCE LEARNING STUDENT SCHEDULE

6	/01/20 to 6/05/20 Week 8 (5 days)				
Directions:	<b>Parents:</b> Assist students with accessing the "Knowledge on the Go" videos, Problem Sets in this packet, and i-Ready through the Clever app. Also, monitor student's progress while working on the videos and/or online lessons.				
Target Standard(s)	<b>Students:</b> Click or watch the "Knowledge on the Go" video each day and complete the daily Problem Set. Visit i-ready to continue your learning path and complete Teacher-Assigned lessons. 7.SP.A.1, 7.SP.A.2				
Module Topic Materials Needed:	<ul> <li>dule 5: Statistics and Probability</li> <li>c C: Random Sampling and Estimating Population Characteristics</li> <li>Access to Knowledge on the Go Lesson Videos &amp; Resources including Templates &amp; Homework Helpers which provide guidance with worked examples for each lesson.</li> <li>Clever Access for i-Ready (see links and QR codes below)</li> <li>Paper, Pencil, Academic Packet including Problem Sets</li> </ul>				





Knowledge on the Go Videos clever.com

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Additional Resources

	Daily Lesson	Extension	Intervention
	(50 minutes)	(10-15 minutes)	(15 minutes)
Day 34	Knowledge on the Go Lesson Materials for Module	i-Ready	i-Ready
	<u>5, Lesson 16</u>	"Teacher Assigned"	"My Path"
		Lesson	Lesson
Day 35	Knowledge on the Go Lesson Materials for Module	i-Ready	i-Ready
	<u>5, Lesson 17</u>	"Teacher Assigned"	"My Path"
		Lesson	Lesson
Day 36	Knowledge on the Go Lesson Materials for Module	i-Ready	i-Ready
	<u>5, Lesson 18</u>	"Teacher Assigned"	"My Path"
		Lesson	Lesson
Day 37	Knowledge on the Go Lesson Materials for Module	i-Ready	i-Ready
	<u>5, Lesson 19</u>	"Teacher Assigned"	"My Path"
		Lesson	Lesson
Day 38	Knowledge on the Go Lesson Materials for Module	i-Ready	i-Ready
	<u>5, Lesson 20</u>	"Teacher Assigned"	"My Path"
		Lesson	Lesson

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

Lesson 34		
Standard	7.SP.A.2	
Learning	Students select a random sample from a population.	
Target	Given a description of a population, students design a plan for selecting a random sample from that population.	
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for Module 5, Lesson 16. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.	
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 5</b> , <b>Lesson 16</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.	
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 5 Lesson 16.	
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.	
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.	

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

Lesson 35		
Standard	7.SP.A.2	
Learning	Students use data from a random sample to estimate a population mean.	
Target	Students understand the term sampling variability in the context of	
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 5</b> , Lesson 17. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.	
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 5</b> , <b>Lesson 17</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.	
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 5 Lesson 17.	
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.	
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.	

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

	Lesson 36	
Standard	7.SP.A.1, 7.SP.A.2	
Learning	Students use data from a random sample to estimate a population mean.	
Target	Students know that increasing the sample size decreases the sampling	
	variability of the sample mean.	
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 5</b> , Lesson 18. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.	
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 5</b> , <b>Lesson 18</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.	
Closing	<b>Recommended</b> : Students will reflect and share their learning on Module 5 Lesson 18.	
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready. SCAN ME	
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.	

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

	Lesson 37	
Standard	7.SP.A.1, 7.SP.A.2	
Learning	Students understand the term sampling variability in the context of	
Target	estimating a population proportion. Students know that increasing the sample size decreases sampling	
Laurah	Variability.	
Launch	Go" video for Module 5, Lesson 19. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.	
	SCAN ME	
Guided	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 5</b> ,	
Practice	<b>Lesson 19</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.	
Closing	Recommended: Students will reflect and share their learning on Module 5 Lesson 19.	
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.	
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.	

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

Lesson 38			
Standard	7.SP.A.2	7.SP.A.2	
Learning	Students use data from a random sample to estimate a population		
Target	proportion.		
Launch	SCAN ME	<b>Recommended:</b> : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 5</b> , Lesson 20. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.	
Guided	Recommend	ed: : Students will complete the Problem Set for Module 5,	
Practice	<i>Lesson 20</i> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.		
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 5 Lesson 20.		
Extend	SCAN ME	<b>Recommended: :</b> Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.	
Intervention	Recommend	ed: : Students will work on their individual Learning Path (My	
	Path) in i-Rea	dy. Visit <u>Clever.com</u> to access i-Ready.	
Click the Knowledge	e on the Golesson	Materials link or scan the Knowledge on the Go OP Code in the Materials sec	

# **Grade 7 Mathematics**

WEEKLY DISTANCE LEARNING STUDENT SCHEDULE

6	/08/20 to 6/12/20 Week 9 (5 days)
Directions:	<b>Parents:</b> Assist students with accessing the "Knowledge on the Go" videos, Problem Sets in this packet, and i-Ready through the Clever app. Also, monitor student's progress while working on the videos and/or online lessons. <b>Students:</b> Click or watch the "Knowledge on the Go" video each day and
	complete the daily Problem Set. Visit i-ready to continue your learning path and complete Teacher-Assigned lessons.
Target Standard(s)	/.SP.B.3, /.SP.B.4, /.G.B.5
Module Topic	Module 5: Statistics and Probability Topic D: Comparing Populations Module 6: Geometry Topic A: Unknown Angles
Materials Needed:	<ul> <li>Access to Knowledge on the Go Lesson Videos &amp; Resources including Templates &amp; Homework Helpers which provide guidance with worked examples for each lesson.</li> <li>Clever Access for i-Ready (see links and QR codes below)</li> <li>Paper, Pencil, Academic Packet including Problem Sets</li> </ul>



	Knowledge on the Go Videos	<u>clever.com</u> <u>Module 5</u>	Module 6
	Daily Lesson	Extension	Intervention
	(50 minutes)	(10-15 minutes)	(15 minutes)
Day 39	Knowledge on the Go Lesson Materials for Module 5,	i-Ready	i-Ready
	Lesson 21	"Teacher Assigned" Lesson	"My Path"
			Lesson
Day 40	Knowledge on the Go Lesson Materials for Module 5,	i-Ready	i-Ready
	Lesson 22	"Teacher Assigned" Lesson	"My Path"
			Lesson
Day 41	Knowledge on the Go Lesson Materials for Module 5,	i-Ready	i-Ready
	Lesson 23	"Teacher Assigned" Lesson	"My Path"
			Lesson
Day 42	Knowledge on the Go Lesson Materials for Module 6,	i-Ready	i-Ready
	Lesson 1	"Teacher Assigned" Lesson	"My Path"
			Lesson
Day 43	Knowledge on the Go Lesson Materials for Module 6,	i-Ready	i-Ready
	Lesson 2	"Teacher Assigned" Lesson	"My Path"
			Lesson

Click the Knowledge on the Go Lesson Materials Link or scan the QR Reader Code in the materials section. Then, scroll down and click on corresponding Module and Lesson.

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

	Lesson 39	
Standard	7.SP.B.3, 7.SP.B.4	
Learning Target	Students understand that a meaningful difference between two sample means is one that is greater than would have been expected due to just sampling variability.	
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 5</b> , Lesson 21. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.	
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 5</b> , <b>Lesson 21</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.	
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 5 Lesson 21.	
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready. SCAN ME	
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.	

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

	Lesson 40	
Standard	7.SP.B.3, 7.SP.B.4	
Learning Target	Students express the difference in sample means as a multiple of a measure of variability. Students understand that a difference in sample means provides evidence that the population means are different if the difference is larger than what would be expected as a result of sampling variability alone.	
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 5</b> , Lesson 22. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.	
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 5</b> , <b>Lesson 22</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.	
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 5 Lesson 22.	
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready. SCAN ME	
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My	

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Lesson 41		
Standard	7.SP.B.3, 7.SP.B.4	
Learning	Students use data from random samples to draw informal inferences about	
Target	the difference in population means.	
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 5</b> , Lesson 23. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.	
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 5</b> , <b>Lesson 23</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.	
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 5 Lesson 23.	
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.	
Intervention	Recommended: : Students will work on their individual Learning Path (My	
	Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.	

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In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

Lesson 42		
Standard	7.G.B.5	
Learning	Students solve for unknown angles in word problems and in diagrams	
Target	involving complementary and supplementary angles.	
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 6</b> , Lesson 1. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.	
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 6</b> , <b>Lesson 1</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.	
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 6 Lesson 1.	
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.	
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.	

#### **Mathematical Fluencies:**

In Grade 7, students are expected to know Fluency with rational number arithmetic. Solve multistep problems with positive and negative rational numbers in any form. Solve one-variable equations of the form px + q = r and p(x + q) = r fluently by end of year. This is a great time to practice these skills.

Standard	7.G.B.5	
Learning Target	Students use data from a random sample to estimate a population proportion. Students solve for unknown angles in word problems and in	
luigei	diagrams involving complementary, supplementary, vertical, and adjacent angles.	
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> Go" video for <b>Module 6</b> , Lesson 2. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.	
Guided	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 6</b> ,	
Practice	<b>Lesson 2</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.	
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 6 Lesson 2.	
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.	
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.	

# **Grade 7 Mathematics**

WEEKLY DISTANCE LEARNING STUDENT SCHEDULE

6/	15/20 to 6/19/20 Week 10 (4 days)		
Directions:	<ul> <li><u>Parents:</u> Assist students with accessing the "Knowledge on the Go" videos, Problem Sets in this packet, and i-Ready through the Clever app. Also, monitor student's progress while working on the videos and/or online lessons.</li> <li><u>Students:</u> Click or watch the "Knowledge on the Go" video each day and complete the daily Problem Set. Visit i-ready to continue your learning path</li> </ul>		
Target Standard(s)	and complete Teacher-Assigned lessons. 7.G.A.2, 7.G.B.5		
Module Topic	Module 6: Geometry Topic A: Unknown Angles Topic B: Constructing Triangles		
Materials Needed:	<ul> <li>Access to Knowledge on the Go Lesson Videos &amp; Resources including Templates &amp; Homework Helpers which provide guidance with worked examples for each lesson.</li> <li>Clever Access for i-Ready (see links and QR codes below)</li> <li>Paper, Pencil, Academic Packet including Problem Sets</li> </ul>		

	Knowledge on the Go Videos	<u>clever.com</u> <u>M</u>	odule 6
	Daily Lesson	Extension	Intervention
	(50 minutes)	(10-15 minutes)	(15 minutes)
Day 44	Knowledge on the Go Lesson Materials for Module 6,	i-Ready	i-Ready
	Lesson 3	"Teacher Assigned"	"My Path"
		Lesson	Lesson
Day 45	Knowledge on the Go Lesson Materials for Module 6,	i-Ready	i-Ready
	Lesson 4	"Teacher Assigned"	"My Path"
		Lesson	Lesson
Day 46	Knowledge on the Go Lesson Materials for Module 6,	i-Ready	i-Ready
	Lesson 5	"Teacher Assigned"	"My Path"
		Lesson	Lesson
Day 47	Knowledge on the Go Lesson Materials for Module 6,	i-Ready	i-Ready
	Lesson 6	"Teacher Assigned"	"My Path"
		Lesson	Lesson

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	Lesson 44		
Standard	7.G.B.5		
Learning	Students solve for unknown angles in word problems and in diagrams		
Target	involving all learned angle facts.		
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 6</b> , Lesson 3. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.		
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 6</b> , <b>Lesson 3</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.		
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 6 Lesson 3.		
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.		
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My		

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	Lesson 45		
Standard	7.G.B.5		
Learning	Students solve for unknown angles in word problems and in diagrams		
Target	involving all learned angle facts.		
Launch	Recommended: : Students will view the "Knowledge on the Go" video for Module 6, Lesson 4. Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.		
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 6</b> , <b>Lesson 4</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.		
Closing	<b>Recommended:</b> Students will reflect and share their learning on Module 6 Lesson 4.		
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.		
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My Path) in i-Ready. Visit Clever.com to access i-Ready.		

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	Lesson 46		
Standard	7.G.B.2		
Learning Target	Students use a triangle correspondence to recognize when two triangles match identically. Students use notation to denote a triangle correspondence and use the triangle correspondence to talk about corresponding angles and sides. Students are able to label equal angles and sides of triangles with multiple arcs or tick marks.		
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 6</b> , <b>Lesson 5</b> . Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.		
Guided Practice	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 6</b> , <b>Lesson 5</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.		
Closing	<b>Recommended: :</b> Students will reflect and share their learning on Module 6 Lesson 5.		
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.		
Intervention	<b>Recommended:</b> : Students will work on their individual Learning Path (My Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.		

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	Lesson 47		
Standard	7.G.B.2		
Learning	Students use a compass, protractor, and ruler to draw geometric shapes		
Target	based on given conditions.		
Launch	Recommended: : Students will view the " <u>Knowledge on the</u> <u>Go</u> " video for <b>Module 6</b> , <b>Lesson 6</b> . Scan the Knowledge on the Go QR Code or click the link to access the video. We encourage parents to assist students with accessing and engaging with the "Knowledge on the Go" videos.		
Guided	<b>Recommended: :</b> Students will complete the Problem Set for <b>Module 6</b> ,		
Practice	<b>Lesson 6</b> from the "Knowledge on the Go" video along with the instructor. These are included in this academic packet.		
Closing	<b>Recommended: :</b> Students will reflect and share their learning on Module 6 Lesson 6.		
Extend	Recommended: : Students will complete the "Teacher Assigned" lesson in i-Ready. Visit <u>Clever.com</u> to access i- Ready.		
Intervention	Recommended: : Students will work on their individual Learning Path (My		
	Path) in i-Ready. Visit <u>Clever.com</u> to access i-Ready.		

Learn, Practice, Succeed

# Eureka Math® Grade 7 Module 4

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G7-M4-LPS-05.2019

- 1. Create a model to represent the following percents.
  - a. 90% b. 0.9% c. 900% d.  $\frac{9}{10}\%$
- 2. Benjamin believes that  $\frac{1}{2}$ % is equivalent to 50%. Is he correct? Why or why not?
- 3. Order the following from least to greatest:  $100\%, \frac{1}{100}, 0.001\%, \frac{1}{10}, 0.001, 1.1, 10, \text{ and } \frac{10,000}{100}$
- 4. Fill in the chart by converting between fractions, decimals, and percents. Show work in the space below.

Fraction	Decimal	Percent
		100%
	0.0825	
	6.25	
		$\frac{1}{8}\%$
$\frac{2}{300}$		
		33.3%
$\frac{\frac{3}{4}}{100}$		
		250%
	0.005	
$\frac{150}{100}$		
	0.055	



- 1. Represent each situation using an equation. Check your answer with a visual model or numeric method.
  - a. What number is 40% of 90?
  - b. What number is 45% of 90?
  - c. 27 is 30% of what number?
  - d. 18 is 30% of what number?
  - e. 25.5 is what percent of 85?
  - f. 21 is what percent of 60?
- 2. 40% of the students on a field trip love the museum. If there are 20 students on the field trip, how many love the museum?
- 3. Maya spent 40% of her savings to pay for a bicycle that cost her \$85.
  - a. How much money was in her savings to begin with?
  - b. How much money does she have left in her savings after buying the bicycle?
- 4. Curtis threw 15 darts at a dartboard. 40% of his darts hit the bull's-eye. How many darts did not hit the bull's-eye?
- 5. A tool set is on sale for \$424.15. The original price of the tool set was \$499.00. What percent of the original price is the sale price?
- 6. Matthew scored a total of 168 points in basketball this season. He scored 147 of those points in the regular season, and the rest were scored in his only playoff game. What percent of his total points did he score in the playoff game?
- 7. Brad put 10 crickets in his pet lizard's cage. After one day, Brad's lizard had eaten 20% of the crickets he had put in the cage. By the end of the next day, the lizard had eaten 25% of the remaining crickets. How many crickets were left in the cage at the end of the second day?
- 8. A furnace used 40% of the fuel in its tank in the month of March and then used 25% of the remaining fuel in the month of April. At the beginning of March, there were 240 gallons of fuel in the tank. How much fuel (in gallons) was left at the end of April?
- 9. In Lewis County, there were 2,277 student athletes competing in spring sports in 2014. That was 110% of the number from 2013, which was 90% of the number from the year before. How many student athletes signed up for a spring sport in 2012?
- 10. Write a real-world word problem that could be modeled by the equation below. Identify the elements of the percent equation and where they appear in your word problem, and then solve the problem.

57.5 = p(250)



- 1. Solve each problem using an equation.
  - a. 49.5 is what percent of 33?
  - b. 72 is what percent of 180?
  - c. What percent of 80 is 90?
- 2. This year, Benny is 12 years old, and his mom is 48 years old.
  - a. What percent of his mom's age is Benny's age?
  - b. What percent of Benny's age is his mom's age?
  - c. In two years, what percent of his age will Benny's mom's age be at that time?
  - d. In 10 years, what percent will Benny's mom's age be of his age?
  - e. In how many years will Benny be 50% of his mom's age?
  - f. As Benny and his mom get older, Benny thinks that the percent of difference between their ages will decrease as well. Do you agree or disagree? Explain your reasoning.
- 3. This year, Benny is 12 years old. His brother Lenny's age is 175% of Benny's age. How old is Lenny?
- 4. When Benny's sister Penny is 24, Benny's age will be 125% of her age.
  - a. How old will Benny be then?
  - b. If Benny is 12 years old now, how old is Penny now? Explain your reasoning.
- 5. Benny's age is currently 200% of his sister Jenny's age. What percent of Benny's age will Jenny's age be in 4 years?



- 6. At an animal shelter, there are 15 dogs, 12 cats, 3 snakes, and 5 parakeets.
  - a. What percent of the number of cats is the number of dogs?
  - b. What percent of the number of cats is the number of snakes?
  - c. What percent less parakeets are there than dogs?
  - d. Which animal has 80% of the number of another animal?
  - e. Which animal makes up approximately 14% of the animals in the shelter?
- 7. Is 2 hours and 30 minutes more or less than 10% of a day? Explain your answer.
- 8. A club's membership increased from 25 to 30 members.
  - a. Express the new membership as a percent of the old membership.
  - b. Express the old membership as a percent of the new membership.
- 9. The number of boys in a school is 120% the number of girls at the school.
  - a. Find the number of boys if there are 320 girls.
  - b. Find the number of girls if there are 360 boys.
- 10. The price of a bicycle was increased from 300 to 450.
  - a. What percent of the original price is the increased price?
  - b. What percent of the increased price is the original price?
- 11. The population of Appleton is 175% of the population of Cherryton.
  - a. Find the population in Appleton if the population in Cherryton is 4,000 people.
  - b. Find the population in Cherryton if the population in Appleton is 10,500 people.

12. A statistics class collected data regarding the number of boys and the number of girls in each classroom at their school during homeroom. Some of their results are shown in the table below.

Number of Boys (x)	Number of Girls ( <i>y</i> )	Number of Girls as a Percent of the Number of Boys
10	5	
	1	25%
18	12	
5	10	
4		50%
20		90%
	10	250%
	6	60%
11		200%
	5	$33\frac{1}{3}\%$
15		20%
	15	75%
6	18	
25	10	
10		110%
	2	10%
16		75%
	7	50%
3		200%
12	10	

a. Complete the blank cells of the table using your knowledge about percent.

- b. Using a coordinate plane and grid paper, locate and label the points representing the ordered pairs (x, y).
- c. Locate all points on the graph that would represent classrooms in which the number of girls y is 100% of the number of boys x. Describe the pattern that these points make.



- d. Which points represent the classrooms in which the number of girls as a percent of the number of boys is greater than 100%? Which points represent the classrooms in which the number of girls as a percent of the number of boys is less than 100%? Describe the locations of the points in relation to the points in part (c).
- e. Find three ordered pairs from your table representing classrooms where the number of girls is the same percent of the number of boys. Do these points represent a proportional relationship? Explain your reasoning.
- f. Show the relationship(s) from part (e) on the graph, and label them with the corresponding equation(s).
- g. What is the constant of proportionality in your equation(s), and what does it tell us about the number of girls and the number of boys at each point on the graph that represents it? What does the constant of proportionality represent in the table in part (a)?



- 1. A store advertises 15% off an item that regularly sells for \$300.
  - a. What is the sale price of the item?
  - b. How is a 15% discount similar to a 15% decrease? Explain.
  - c. If 8% sales tax is charged on the sale price, what is the total with tax?
  - d. How is 8% sales tax like an 8% increase? Explain.
- 2. An item that was selling for \$72.00 is reduced to \$60.00. Find the percent decrease in price. Round your answer to the nearest tenth.
- 3. A baseball team had 80 players show up for tryouts last year and this year had 96 players show up for tryouts. Find the percent increase in players from last year to this year.
- 4. At a student council meeting, there was a total of 60 students present. Of those students, 35 were female.
  - a. By what percent is the number of females greater than the number of males?
  - b. By what percent is the number of males less than the number of females?
  - c. Why is the percent increase and percent decrease in parts (a) and (b) different?
- 5. Once each day, Darlene writes in her personal diary and records whether the sun is shining or not. When she looked back though her diary, she found that over a period of 600 days, the sun was shining 60% of the time. She kept recording for another 200 days and then found that the total number of sunny days dropped to 50%. How many of the final 200 days were sunny days?
- 6. Henry is considering purchasing a mountain bike. He likes two bikes: One costs \$500, and the other costs \$600. He tells his dad that the bike that is more expensive is 20% more than the cost of the other bike. Is he correct? Justify your answer.
- 7. State two numbers such that the lesser number is 25% less than the greater number.
- 8. State two numbers such that the greater number is 75% more than the lesser number.
- 9. Explain the difference in your thought process for Problems 7 and 8. Can you use the same numbers for each problem? Why or why not?



10. In each of the following expressions, *c* represents the original cost of an item.

- i. 0.90*c*
- ii. 0.10*c*
- iii. c 0.10c
- a. Circle the expression(s) that represents 10% of the original cost. If more than one answer is correct, explain why the expressions you chose are equivalent.
- b. Put a box around the expression(s) that represents the final cost of the item after a 10% decrease. If more than one is correct, explain why the expressions you chose are equivalent.
- c. Create a word problem involving a percent decrease so that the answer can be represented by expression (ii).
- d. Create a word problem involving a percent decrease so that the answer can be represented by expression (i).
- e. Tyler wants to know if it matters if he represents a situation involving a 25% decrease as 0.25x or (1 0.25)x. In the space below, write an explanation that would help Tyler understand how the context of a word problem often determines how to represent the situation.


Use a double number line to answer Problems 1–5.

- 1. Tanner collected 360 cans and bottles while fundraising for his baseball team. This was 40% of what Reggie collected. How many cans and bottles did Reggie collect?
- 2. Emilio paid \$287.50 in taxes to the school district that he lives in this year. This year's taxes were a 15% increase from last year. What did Emilio pay in school taxes last year?
- 3. A snowmobile manufacturer claims that its newest model is 15% lighter than last year's model. If this year's model weighs 799 lb., how much did last year's model weigh?
- 4. Student enrollment at a local school is concerning the community because the number of students has dropped to 504, which is a 20% decrease from the previous year. What was the student enrollment the previous year?
- 5. The color of paint used to paint a race car includes a mixture of yellow and green paint. Scotty wants to lighten the color by increasing the amount of yellow paint 30%. If a new mixture contains 3.9 liters of yellow paint, how many liters of yellow paint did he use in the previous mixture?

Use factors of 100 and mental math to answer Problems 6–10. Describe the method you used.

- 6. Alexis and Tasha challenged each other to a typing test. Alexis typed 54 words in one minute, which was 120% of what Tasha typed. How many words did Tasha type in one minute?
- 7. Yoshi is 5% taller today than she was one year ago. Her current height is 168 cm. How tall was she one year ago?
- 8. Toya can run one lap of the track in 1 min. 3 sec., which is 90% of her younger sister Niki's time. What is Niki's time for one lap of the track?
- 9. An animal shelter houses only cats and dogs, and there are 25% more cats than dogs. If there are 40 cats, how many dogs are there, and how many animals are there total?
- 10. Angle scored 91 points on a test but only received a 65% grade on the test. How many points were possible on the test?

For Problems 11–17, find the answer using any appropriate method.

- 11. Robbie owns 15% more movies than Rebecca, and Rebecca owns 10% more movies than Joshua. If Rebecca owns 220 movies, how many movies do Robbie and Joshua each have?
- 12. 20% of the seventh-grade students have math class in the morning.  $16\frac{2}{3}$ % of those students also have science class in the morning. If 30 seventh-grade students have math class in the morning but not science class, find how many seventh-grade students there are.



- 13. The school bookstore ordered three-ring notebooks. They put 75% of the order in the warehouse and sold 80% of the rest in the first week of school. There are 25 notebooks left in the store to sell. How many three-ring notebooks did they originally order?
- 14. In the first game of the year, the modified basketball team made 62.5% of their foul shot free throws. Matthew made all 6 of his free throws, which made up 25% of the team's free throws. How many free throws did the team miss altogether?
- 15. Aiden's mom calculated that in the previous month, their family had used 40% of their monthly income for gasoline, and 63% of that gasoline was consumed by the family's SUV. If the family's SUV used \$261.45 worth of gasoline last month, how much money was left after gasoline expenses?
- 16. Rectangle A is a scale drawing of Rectangle B and has 25% of its area. If Rectangle A has side lengths of 4 cm and 5 cm, what are the side lengths of Rectangle B?



17. Ted is a supervisor and spends 20% of his typical work day in meetings and 20% of that meeting time in his daily team meeting. If he starts each day at 7:30 a.m., and his daily team meeting is from 8:00 a.m. to 8:20 a.m., when does Ted's typical work day end?



- 1. Micah has 294 songs stored in his phone, which is 70% of the songs that Jorge has stored in his phone. How many songs are stored on Jorge's phone?
- 2. Lisa sold 81 magazine subscriptions, which is 27% of her class's fundraising goal. How many magazine subscriptions does her class hope to sell?
- 3. Theresa and Isaiah are comparing the number of pages that they read for pleasure over the summer. Theresa read 2,210 pages, which was 85% of the number of pages that Isaiah read. How many pages did Isaiah read?
- 4. In a parking garage, the number of SUVs is 40% greater than the number of non-SUVs. Gina counted 98 SUVs in the parking garage. How many vehicles were parked in the garage?
- 5. The price of a tent was decreased by 15% and sold for \$76.49. What was the original price of the tent in dollars?
- 6. 40% of the students at Rockledge Middle School are musicians. 75% of those musicians have to read sheet music when they play their instruments. If 38 of the students can play their instruments without reading sheet music, how many students are there at Rockledge Middle School?
- 7. At Longbridge Middle School, 240 students said that they are an only child, which is 48% of the school's student enrollment. How many students attend Longbridge Middle School?
- 8. Grace and her father spent  $4\frac{1}{2}$  hours over the weekend restoring their fishing boat. This time makes up 6% of the time needed to fully restore the boat. How much total time is needed to fully restore the boat?
- 9. Bethany's mother was upset with her because Bethany's text messages from the previous month were 218% of the amount allowed at no extra cost under her phone plan. Her mother had to pay for each text message over the allowance. Bethany had 5,450 text messages last month. How many text messages is she allowed under her phone plan at no extra cost?
- 10. Harry used 84% of the money in his savings account to buy a used dirt bike that cost him \$1,050. How much money is left in Harry's savings account?
- 11. 15% of the students in Mr. Riley's social studies classes watch the local news every night. Mr. Riley found that 136 of his students do not watch the local news. How many students are in Mr. Riley's social studies classes?
- 12. Grandma Bailey and her children represent about 9.1% of the Bailey family. If Grandma Bailey has 12 children, how many members are there in the Bailey family?
- 13. Shelley earned 20% more money in tips waitressing this week than last week. This week she earned \$72.00 in tips waitressing. How much money did Shelley earn last week in tips?



- 14. Lucy's savings account has 35% more money than her sister Edy's. Together, the girls have saved a total of \$206.80. How much money has each girl saved?
- 15. Bella spent 15% of her paycheck at the mall, and 40% of that was spent at the movie theater. Bella spent a total of \$13.74 at the movie theater for her movie ticket, popcorn, and a soft drink. How much money was in Bella's paycheck?
- 16. On a road trip, Sara's brother drove 47.5% of the trip, and Sara drove 80% of the remainder. If Sara drove for 4 hours and 12 minutes, how long was the road trip?



- 1. You have a coupon for an additional 25% off the price of any sale item at a store. The store has put a robotics kit on sale for 15% off the original price of \$40. What is the price of the robotics kit after both discounts?
- 2. A sign says that the price marked on all music equipment is 30% off the original price. You buy an electric guitar for the sale price of \$315.
  - a. What is the original price?
  - b. How much money did you save off the original price of the guitar?
  - c. What percent of the original price is the sale price?
- 3. The cost of a New York Yankee baseball cap is \$24.00. The local sporting goods store sells it for \$30.00. Find the markup rate.
- 4. Write an equation to determine the selling price in dollars, *p*, on an item that is originally priced *s* dollars after a markdown of 15%.
  - a. Create and label a table showing five possible pairs of solutions to the equation.
  - b. Create and label a graph of the equation.

- c. Interpret the points (0,0) and (1,r).
- 5. At the amusement park, Laura paid \$6.00 for a small cotton candy. Her older brother works at the park, and he told her they mark up the cotton candy by 300%. Laura does not think that is mathematically possible. Is it possible, and if so, what is the price of the cotton candy before the markup?



- 6. A store advertises that customers can take 25% off the original price and then take an extra 10% off. Is this the same as a 35% off discount? Explain.
- 7. An item that costs \$50.00 is marked 20% off. Sales tax for the item is 8%. What is the final price, including tax?
  - a. Solve the problem with the discount applied before the sales tax.
  - b. Solve the problem with the discount applied after the sales tax.
  - c. Compare your answers in parts (a) and (b). Explain.
- 8. The sale price for a bicycle is \$315. The original price was first discounted by 50% and then discounted an additional 10%. Find the original price of the bicycle.
- 9. A ski shop has a markup rate of 50%. Find the selling price of skis that cost the storeowner \$300.
- 10. A tennis supply store pays a wholesaler \$90 for a tennis racquet and sells it for \$144. What is the markup rate?
- 11. A shoe store is selling a pair of shoes for \$60 that has been discounted by 25%. What was the original selling price?
- 12. A shoe store has a markup rate of 75% and is selling a pair of shoes for \$133. Find the price the store paid for the shoes.
- 13. Write  $5\frac{1}{4}$ % as a simple fraction.
- 14. Write  $\frac{3}{8}$  as a percent.
- 15. If 20% of the 70 faculty members at John F. Kennedy Middle School are male, what is the number of male faculty members?
- 16. If a bag contains 400 coins, and  $33\frac{1}{2}\%$  are nickels, how many nickels are there? What percent of the coins are not nickels?
- 17. The temperature outside is 60 degrees Fahrenheit. What would be the temperature if it is increased by 20%?

- 1. The odometer in Mr. Washington's car does not work correctly. The odometer recorded 13.2 miles for his last trip to the hardware store, but he knows the distance traveled is 15 miles. What is the percent error? Use a calculator and the percent error formula to help find the answer. Show your steps.
- The actual length of a soccer field is 500 feet. A measuring instrument shows the length to be 493 feet. The actual width of the field is 250 feet, but the recorded width is 246.5 feet. Answer the following questions based on this information. Round all decimals to the nearest tenth.
  - a. Find the percent error for the length of the soccer field.
  - b. Find the percent error of the area of the soccer field.
  - c. Explain why the values from parts (a) and (b) are different.



500 feet

- 3. Kayla's class went on a field trip to an aquarium. One tank had 30 clown fish. She miscounted the total number of clown fish in the tank and recorded it as 24 fish. What is Kayla's percent error?
- 4. Sid used geometry software to draw a circle of radius 4 units on a grid. He estimated the area of the circle by counting the squares that were mostly inside the circle and got an answer of 52 square units.



- a. Is his estimate too large or too small?
- b. Find the percent error in Sid's estimation to the nearest hundredth using the  $\pi$  key on your calculator.
- 5. The exact value for the density of aluminum is 2.699 g/cm<sup>3</sup>. Working in the science lab at school, Joseph finds the density of a piece of aluminum to be 2.75 g/cm<sup>3</sup>. What is Joseph's percent error? (Round to the nearest hundredth.)
- 6. The world's largest marathon, The New York City Marathon, is held on the first Sunday in November each year. Between 2 million and 2.5 million spectators will line the streets to cheer on the marathon runners. At most, what is the percent error?



- 7. A circle is inscribed inside a square, which has a side length of 12.6 cm. Jared estimates the area of the circle to be about 80% of the area of the square and comes up with an estimate of  $127 \text{ cm}^2$ .
  - a. Find the absolute error from Jared's estimate to two decimal places using the  $\pi$  key on your calculator.
  - b. Find the percent error of Jared's estimate to two decimal places using the  $\pi$  key on your calculator.
  - c. Do you think Jared's estimate was reasonable?
  - d. Would this method of computing the area of a circle always be too large?
- 8. In a school library, 52% of the books are paperback. If there are 2,658 books in the library, how many of them are not paperback to the nearest whole number?
- 9. Shaniqua has 25% less money than her older sister Jennifer. If Shaniqua has \$180, how much money does Jennifer have?
- 10. An item that was selling for \$1,102 is reduced to \$806. To the nearest whole, what is the percent decrease?
- 11. If 60 calories from fat is 75% of the total number of calories in a bag of chips, find the total number of calories in the bag of chips.



12.6 cm



- 1. Solve each problem using an equation.
  - a. What is 150% of 625?
  - b. 90 is 40% of what number?
  - c. What percent of 520 is 40? Round to the nearest hundredth of a percent.
- 2. The actual length of a machine is 12.25 cm. The measured length is 12.2 cm. Round the answer to part (b) to the nearest hundredth of a percent.
  - a. Find the absolute error.
  - b. Find the percent error.
- 3. A rowing club has 600 members. 60% of them are women. After 200 new members joined the club, the percentage of women was reduced to 50%. How many of the new members are women?
- 4. 40% of the marbles in a bag are yellow. The rest are orange and green. The ratio of the number of orange to the number of green is 4:5. If there are 30 green marbles, how many yellow marbles are there? Use a visual model to show your answer.
- 5. Susan has 50% more books than Michael. Michael has 40 books. If Michael buys 8 more books, will Susan have more or less books than Michael? What percent more or less will Susan's books be? Use any method to solve the problem.
- 6. Harry's amount of money is 75% of Kayla's amount of money. After Harry earned \$30 and Kayla earned 25% more of her money, Harry's amount of money is 80% of Kayla's money. How much money did each have at the beginning? Use a visual model to solve the problem.



- 1. Enrique takes out a student loan to pay for his college tuition this year. Find the interest on the loan if he borrowed \$2,500 at an annual interest rate of 6% for 15 years.
- 2. Your family plans to start a small business in your neighborhood. Your father borrows \$10,000 from the bank at an annual interest rate of 8% rate for 36 months. What is the amount of interest he will pay on this loan?
- 3. Mr. Rodriguez invests \$2,000 in a savings plan. The savings account pays an annual interest rate of 5.75% on the amount he put in at the end of each year.
  - a. How much will Mr. Rodriguez earn if he leaves his money in the savings plan for 10 years?
  - b. How much money will be in his savings plan at the end of 10 years?
  - c. Create (and label) a graph in the coordinate plane to show the relationship between time and the amount of interest earned for 10 years. Is the relationship proportional? Why or why not? If so, what is the constant of proportionality?
  - d. Explain what the points (0,0) and (1,115) mean on the graph.
  - e. Using the graph, find the balance of the savings plan at the end of seven years.
  - f. After how many years will Mr. Rodriguez have increased his original investment by more than 50%? Show your work to support your answer.

### **Challenge Problem**

4. George went on a game show and won \$60,000. He wanted to invest it and found two funds that he liked. Fund 250 earns 15% interest annually, and Fund 100 earns 8% interest annually. George does not want to earn more than \$7,500 in interest income this year. He made the table below to show how he could invest the money.

	Ι	Р	r	t
Fund 100		X	0.08	1
Fund 250		60,000 – <i>x</i>	0.15	1
Total	7,500	60,000		

- a. Explain what value *x* is in this situation.
- b. Explain what the expression 60,000 x represents in this situation.
- c. Using the simple interest formula, complete the table for the amount of interest earned.
- d. Write an inequality to show the total amount of interest earned from both funds.
- e. Use algebraic properties to solve for *x* and the principal, in dollars, George could invest in Fund 100. Show your work.
- f. Use your answer from part (e) to determine how much George could invest in Fund 250.
- g. Using your answers to parts (e) and (f), how much interest would George earn from each fund?



- A school district's property tax rate rises from 2.5% to 2.7% to cover a \$300,000 budget deficit (shortage of money). What is the value of the property in the school district to the nearest dollar? (Note: Property is assessed at 100% of its value.)
- 2. Jake's older brother Sam has a choice of two summer jobs. He can either work at an electronics store or at the school's bus garage. The electronics store would pay him to work 15 hours per week. He would make \$8 per hour plus a 2% commission on his electronics sales. At the school's bus garage, Sam could earn \$300 per week working 15 hours cleaning buses. Sam wants to take the job that pays him the most. How much in electronics would Sam have to sell for the job at the electronics store to be the better choice for his summer job?
- 3. Sarah lost her science book. Her school charges a lost book fee equal to 75% of the cost of the book. Sarah received a notice stating she owed the school \$60 for the lost book.
  - a. Write an equation to represent the proportional relationship between the school's cost for the book and the amount a student must pay for a lost book. Let *B* represent the school's cost of the book in dollars and *N* represent the student's cost in dollars.
  - b. What is the constant or proportionality? What does it mean in the context of this situation?
  - c. How much did the school pay for the book?
- 4. In the month of May, a certain middle school has an average daily absentee rate of 8% each school day. The absentee rate is the percent of students who are absent from school each day.
  - a. Write an equation that shows the proportional relationship between the number of students enrolled in the middle school and the average number of students absent each day during the month of May. Let *s* represent the number of students enrolled in school, and let *a* represent the average number of students absent each day in May.
  - b. Use your equation to complete the table. List 5 possible values for *s* and *a*.



- c. Identify the constant of proportionality, and explain what it means in the context of this situation.
- d. Based on the absentee rate, determine the number of students absent on average from school during the month of May if there are 350 students enrolled in the middle school.



5. The equation shown in the box below could relate to many different percent problems. Put an X next to each problem that could be represented by this equation. For any problem that does not match this equation, explain why it does not.  $Quantity = 1.05 \cdot Whole$ 

Find the amount of an investment after 1 year with 0.5% interest paid annually. Write an equation to show the amount paid for an item including tax, if the tax rate is 5%. A proportional relationship has a constant of proportionality equal to 105%. 0 Whole 100 200 300 400 500 0 105 210 315 420 Quantity 525

Mr. Hendrickson sells cars and earns a 5% commission on every car he sells. Write an equation to show the relationship between the price of a car Mr. Hendrickson sold and the amount of commission he earns.

Lesson 11:

Tax, Commissions, Fees, and Other Real-World Percent Problems



1. Use the diagram below to create a scale drawing using a scale factor of  $133\frac{1}{3}\%$ . Write numerical equations to find the horizontal and vertical distances in the scale drawing.



2. Create a scale drawing of the original drawing given below using a horizontal scale factor of 80% and a vertical scale factor of 175%. Write numerical equations to find the horizontal and vertical distances.



3. The accompanying diagram shows that the length of a pencil from its eraser to its tip is 7 units and that the eraser is 1.5 units wide. The picture was placed on a photocopy machine and reduced to  $66\frac{2}{3}\%$ . Find the new size of the pencil, and sketch a drawing. Write numerical equations to find the new dimensions.





- 4. Use the diagram to answer each question.
  - a. What are the corresponding horizontal and vertical distances in a scale drawing if the scale factor is 25%? Use numerical equations to find your answers.



- b. What are the corresponding horizontal and vertical distances in a scale drawing if the scale factor is 160%? Use a numerical equation to find your answers.
- 5. Create a scale drawing of the original drawing below using a horizontal scale factor of 200% and a vertical scale factor of 250%.





6. Using the diagram below, on grid paper sketch the same drawing using a horizontal scale factor of 50% and a vertical scale factor of 150%.





1. The scale factor from Drawing 1 to Drawing 2 is  $41\frac{2}{3}$ %. Justify why Drawing 1 is a scale drawing of Drawing 2 and why it is an enlargement of Drawing 2. Include the scale factor in your justification.



2. The scale factor from Drawing 1 to Drawing 2 is 40%, and the scale factor from Drawing 2 to Drawing 3 is 37.5%. What is the scale factor from Drawing 1 to Drawing 3? Explain your reasoning, and check your answer using an example.





- 3. Traci took a photograph and printed it to be a size of 4 units by 4 units as indicated in the diagram. She wanted to enlarge the original photograph to a size of 5 units by 5 units and 10 units by 10 units.
  - a. Sketch the different sizes of photographs.



- b. What was the scale factor from the original photo to the photo that is 5 units by 5 units?
- c. What was the scale factor from the original photo to the photo that is 10 units by 10 units?
- d. What was the scale factor from the  $5 \times 5$  photo to the  $10 \times 10$  photo?
- e. Write an equation to verify how the scale factor from the original photo to the enlarged  $10 \times 10$  photo can be calculated using the scale factors from the original to the 5 × 5, and then from the 5 × 5 to the  $10 \times 10$ .
- 4. The scale factor from Drawing 1 to Drawing 2 is 30%, and the scale factor from Drawing 1 to Drawing 3 is 175%. What are the scale factors of each given relationship? Then, answer the question that follows. Drawings are not to scale.
  - a. Drawing 2 to Drawing 3
  - b. Drawing 3 to Drawing 1
  - c. Drawing 3 to Drawing 2
  - d. How can you check your answers?





1. The smaller train is a scale drawing of the larger train. If the length of the tire rod connecting the three tires of the larger train, as shown below, is 36 inches, write an equation to find the length of the tire rod of the smaller train. Interpret your solution in the context of the problem.



2. The larger arrow is a scale drawing of the smaller arrow. The distance around the smaller arrow is 25.66 units. What is the distance around the larger arrow? Use an equation to find the distance and interpret your solution in the context of the problem.





3. The smaller drawing below is a scale drawing of the larger. The distance around the larger drawing is 39.4 units. Using an equation, find the distance around the smaller drawing.



4. The figure is a diagram of a model rocket and is a two-dimensional scale drawing of an actual rocket. The length of a model rocket is 2.5 feet, and the wing span is 1.25 feet. If the length of an actual rocket is 184 feet, use an equation to find the wing span of the actual rocket.





- 1. What percent of the area of the larger circle is shaded?
  - a. Solve this problem using scale factors.
  - b. Verify your work in part (a) by finding the actual areas.

- 2. The area of the large disk is 50.24 units<sup>2</sup>.
  - a. Find the area of the shaded region using scale factors. Use  $3.14\ \text{as}$ an estimate for  $\pi$ .
  - b. What percent of the large circular region is unshaded?

3. Ben cut the following rockets out of cardboard. The height from the base to the tip of the smaller rocket is 20 cm. The height from the base to the tip of the larger rocket is 120 cm. What percent of the area of the smaller rocket is the area of the larger rocket?









4. In the photo frame depicted below, three 5 inch by 5 inch squares are cut out for photographs. If these cut-out regions make up  $\frac{3}{16}$  of the area of the entire photo frame, what are the dimensions of the photo frame?



5. Kelly was online shopping for envelopes for party invitations and saw these images on a website.





The website listed the dimensions of the small envelope as 6 in. by 8 in. and the medium envelope as 10 in. by  $13\frac{1}{3}$  in.

- a. Compare the dimensions of the small and medium envelopes. If the medium envelope is a scale drawing of the small envelope, what is the scale factor?
- b. If the large envelope was created based on the dimensions of the small envelope using a scale factor of 250%, find the dimensions of the large envelope.
- c. If the medium envelope was created based on the dimensions of the large envelope, what scale factor was used to create the medium envelope?
- d. What percent of the area of the larger envelope is the area of the medium envelope?



- 1. One container is filled with a mixture that is 30% acid. A second container is filled with a mixture that is 50% acid. The second container is 50% larger than the first, and the two containers are emptied into a third container. What percent of acid is in the third container?
- 2. The store's markup on a wholesale item is 40%. The store is currently having a sale, and the item sells for 25% off the retail price. What is the percent of profit made by the store?
- During lunch hour at a local restaurant, 90% of customers order a meat entrée and 10% order a vegetarian entrée.
   Of the customers who order a meat entrée, 80% order a drink. Of the customers who order a vegetarian entrée, 40% order a drink. What is the percent of customers who order a drink with their entrée?
- 4. Last year's spell-a-thon spelling test for a first grade class had 15% more words with four or more letters than this year's spelling test. Next year, there will be 5% less than this year. What percent more words have four or more letters in last year's test than next year's?
- 5. An ice cream shop sells 75% less ice cream in December than in June. Twenty percent more ice cream is sold in July than in June. By what percent did ice cream sales increase from December to July?
- 6. The livestock on a small farm the prior year consisted of 40% goats, 10% cows, and 50% chickens. This year, there is a 5% decrease in goats, 9% increase in cows, and 15% increase in chickens. What is the percent increase or decrease of livestock this year?
- 7. In a pet shelter that is occupied by 55% dogs and 45% cats, 60% of the animals are brought in by concerned people who found these animals in the streets. If 90% of the dogs are brought in by concerned people, what is the percent of cats that are brought in by concerned people?
- 8. An artist wants to make a particular teal color paint by mixing a 75% blue hue and 25% yellow hue. He mixes a blue hue that has 85% pure blue pigment and a yellow hue that has 60% of pure yellow pigment. What is the percent of pure pigment that is in the resulting teal color paint?
- 9. On Mina's block, 65% of her neighbors do not have any pets, and 35% of her neighbors own at least one pet. If 25% of the neighbors have children but no pets, and 60% of the neighbors who have pets also have children, what percent of the neighbors have children?



- 1. A 5-liter cleaning solution contains 30% bleach. A 3-liter cleaning solution contains 50% bleach. What percent of bleach is obtained by putting the two mixtures together?
- 2. A container is filled with 100 grams of bird feed that is 80% seed. How many grams of bird feed containing 5% seed must be added to get bird feed that is 40% seed?
- 3. A container is filled with 100 grams of bird feed that is 80% seed. Tom and Sally want to mix the 100 grams with bird feed that is 5% seed to get a mixture that is 40% seed. Tom wants to add 114 grams of the 5% seed, and Sally wants to add 115 grams of the 5% seed mix. What will be the percent of seed if Tom adds 114 grams? What will be the percent of seed if Sally adds 115 grams? How much do you think should be added to get 40% seed?
- 4. Jeanie likes mixing leftover salad dressings together to make new dressings. She combined 0.55 L of a 90% vinegar salad dressing with 0.45 L of another dressing to make 1 L of salad dressing that is 60% vinegar. What percent of the second salad dressing was vinegar?
- 5. Anna wants to make 30 mL of a 60% salt solution by mixing together a 72% salt solution and a 54% salt solution. How much of each solution must she use?
- 6. A mixed bag of candy is 25% chocolate bars and 75% other filler candy. Of the chocolate bars, 50% of them contains caramel. Of the other filler candy, 10% of them contain caramel. What percent of candy contains caramel?
- 7. A local fish market receives the daily catch of two local fishermen. The first fisherman's catch was 84% fish while the rest was other non-fish items. The second fisherman's catch was 76% fish while the rest was other non-fish items. If the fish market receives 75% of its catch from the first fisherman and 25% from the second, what was the percent of other non-fish items the local fish market bought from the fishermen altogether?



# **Opening Exercise**

You are about to switch out your books from your locker during passing period but forget the order of your locker combination. You know that there are the numbers 3, 16, and 21 in some order. What is the percent of locker combinations that start with 3?

Locker Combination	Possibilities:
--------------------	----------------

3, 16, 21						
21, 16, 3						
16, 21, 3						
21, 3, 16						
16, 3, 21						
3, 21, 16						

## Example 1

All of the 3-letter passwords that can be formed using the letters A and B are as follows: AAA, AAB, ABA, ABB, BAA, BAB, BBA, BBB.

a. What percent of passwords contain at least two B's?

b. What percent of passwords contain no A's?



# Exercises 1–2

1. How many 4-letter passwords can be formed using the letters A and B?

- 2. What percent of the 4-letter passwords contain
  - a. No A's?
  - b. Exactly one A?
  - c. Exactly two A's?
  - d. Exactly three A's?
  - e. Four A's?
  - f. The same number of A's and B's?



240

### Example 2

In a set of 3-letter passwords, 40% of the passwords contain the letter B and two of another letter. Which of the two sets below meets the criteria? Explain how you arrived at your answer.

	Set 1		S	et 2
BBB	AAA	CAC	CEB	BBB
CBC	ABA	CCC	EBE	CCC
BBC	CCB	CAB	CCC	EEE
AAB	AAC	BAA	EEB	CBC
ACB	BAC	BCC	CCB	ECE

### Exercises 3–4

3. Shana read the following problem:

"How many letter arrangements can be formed from the word *triangle* that have two vowels and two consonants (order does not matter)?"

She answered that there are 30 letter arrangements.

Twenty percent of the letter arrangements that began with a vowel actually had an English definition. How many letter arrangements that begin with a vowel have an English definition?



- - a. From the list above, what is the percent of melodies with all three notes that are different?
  - b. From the list above, what is the percent of melodies that have three of the same notes?

### Example 3

Look at the 36 points on the coordinate plane with whole number coordinates between 1 and 6, inclusive.



a. Draw a line through each of the points which have an *x*-coordinate and *y*-coordinate sum of 7. Draw a line through each of the points which have an *x*-coordinate and *y*-coordinate sum of 6. Draw a line through each of the points which have an *x*-coordinate and *y*-coordinate sum of 5. Draw a line through each of the points which have an *x*-coordinate and *y*-coordinate sum of 4. Draw a line through each of the points which have an *x*-coordinate and *y*-coordinate sum of 3. Draw a line through each of the points which have an *x*-coordinate and *y*-coordinate sum of 3. Draw a line through each of the points which have an *x*-coordinate and *y*-coordinate sum of 2. Draw a line through each of the points which have an *x*-coordinate and *y*-coordinate sum of 8. Draw a line through each of the points which have an *x*-coordinate and *y*-coordinate sum of 9. Draw a line through each of the points which have an *x*-coordinate and *y*-coordinate sum of 10. Draw a line through each of the points which have an *x*-coordinate and *y*-coordinate sum of 11. Draw a line through each of the points which have an *x*-coordinate and *y*-coordinate sum of 11.



- b. What percent of the 36 points have coordinate sum 7?
- c. Write a numerical expression that could be used to determine the percent of the 36 points that have a coordinate sum of 7.
- d. What percent of the 36 points have coordinate sum 5 or less?
- e. What percent of the 36 points have coordinate sum 4 or 10?



### **Lesson Summary**

To find the percent of possible outcomes for a counting problem you need to determine the total number of possible outcomes and the different favorable outcomes. The representation

 $Quantity = Percent \times Whole$ 

can be used where the quantity is the number of different favorable outcomes and the whole is the total number of possible outcomes.



**Counting Problems** 

Lesson 18:

Name \_

There are a van and a bus transporting students on a student camping trip. Arriving at the site, there are 3 parking spots. Let v represent the van and b represent the bus. The chart shows the different ways the vehicles can park.

a. In what percent of the arrangements are the vehicles separated by an empty parking space?

b. In what percent of the arrangements are the vehicles parked next to each other?

c. In what percent of the arrangements does the left or right parking space remain vacant?





245

Lesson 18 Exit Ticket 7•4

Date \_\_\_\_\_

1. A carnival game requires you to roll a six-sided number cube two times. To determine if you win a prize, you must calculate the product of the two rolls. The possible products are

1	2	3	4	5	6
2	4	6	8	10	12
3	6	9	12	15	18
4	8	12	16	20	24
5	10	15	20	25	30
6	12	18	24	30	36

a. What is the percent that the product will be greater than 20?

I know the numerator represents the number of outcomes with a product greater than 20, and the denominator represents the total number of outcomes. Greater than 20 does not include 20.

b. In order to win the carnival game, the product can be no more than 10. What percent chance do you have of winning the game?



$$\frac{19}{36} = 52\frac{7}{9}\%$$

I have about a 53% chance of winning the carnival game.



2. Calleigh loves to accessorize. She always wears three pieces of jewelry using combinations of rings, necklaces, and bracelets. The table shows the different combinations of accessories Calleigh wore with her last eight outfits.

R	В	Ν	R	В	N	Ν	В
R	Ν	В	В	Ν	R	R	В
R	R	N	В	R	В	В	В

a. What percent of Calleigh's outfits included at least one ring?



b. What percent of Calleigh's outfits only included one type of accessory?

$$\frac{2}{8} = \frac{1}{4} = 25\%$$
The first and last outfit Calleigh wore only included one type of accessory.

- 1. A six-sided die (singular for dice) is thrown twice. The different rolls are as follows:
  - 1 and 1, 1 and 2, 1 and 3, 1 and 4, 1 and 5, 1 and 6,
  - 2 and 1, 2 and 2, 2 and 3, 2 and 4, 2 and 5, 2 and 6, 3 and 1, 3 and 2, 3 and 3, 3 and 4, 3 and 5, 3 and 6, 4 and 1, 4 and 2, 4 and 3, 4 and 4, 4 and 5, 4 and 6,
  - 5 and 1, 5 and 2, 5 and 3, 5 and 4, 5 and 5, 5 and 6,
  - 6 and 1, 6 and 2, 6 and 3, 6 and 4, 6 and 5, 6 and 6.
  - a. What is the percent that both throws will be even numbers?
  - b. What is the percent that the second throw is a 5?
  - c. What is the percent that the first throw is lower than a 6?
- 2. You have the ability to choose three of your own classes, art, language, and physical education. There are three art classes (A1, A2, A3), two language classes (L1, L2), and two P.E. classes (P1, P2) to choose from. The order does not matter and you must choose one from each subject.

A1, L1, P1	A2, L1, P1	A3, L1, P1
A1, L1, P2	A2, L1, P2	A3, L1, P2
A1, L2, P1	A2, L2, P1	A3, L2, P1
A1, L2, P2	A2, L2, P2	A3, L2, P2

Compare the percent of possibilities with A1 in your schedule to the percent of possibilities with L1 in your schedule.

3. Fridays are selected to show your school pride. The colors of your school are orange, blue, and white, and you can show your spirit by wearing a top, a bottom, and an accessory with the colors of your school. During lunch, 11 students are chosen to play for a prize on stage. The table charts what the students wore.

Тор	W	0	W	0	В	W	В	В	W	W	W
Bottom	В	0	В	В	0	В	В	В	0	W	В
Accessory	W	0	В	W	В	0	В	W	0	0	0

- a. What is the percent of outfits that are one color?
- b. What is the percent of outfits that include orange accessories?



4. Shana wears two rings (G represents gold, and S represents silver) at all times on her hand. She likes fiddling with them and places them on different fingers (pinky, ring, middle, index) when she gets restless. The chart is tracking the movement of her rings.

	Pinky Finger	Ring Finger	Middle Finger	Index Finger
Position 1		G	S	
Position 2			S	G
Position 3	G		S	
Position 4				S,G
Position 5	S	G		
Position 6	G	S		
Position 7	S		G	
Position 8	G		S	
Position 9		S,G		
Position 10		G	S	
Position 11			G	S
Position 12		S		G
Position 13	S,G			
Position 14			S,G	

- a. What percent of the positions shows the gold ring on her pinky finger?
- b. What percent of the positions shows both rings on the same finger?
- 5. Use the coordinate plane below to answer the following questions.



- a. What is the percent of the 36 points whose quotient of  $\frac{x \text{coordinate}}{y \text{coordinate}}$  is greater than one?
- b. What is the percent of the 36 points whose coordinate quotient is equal to one?

250

Learn, Practice, Succeed

# Eureka Math<sup>®</sup> Grade 7 Module 5
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1. Match each spinner below with the words *impossible, unlikely, equally likely to occur or not occur, likely,* and *certain* to describe the chance of the spinner landing on black.



- 2. Decide if each of the following events is *impossible, unlikely, equally likely to occur or not occur, likely,* or *certain* to occur.
  - a. A vowel will be picked when a letter is randomly selected from the word *lieu*.
  - b. A vowel will be picked when a letter is randomly selected from the word *math*.
  - c. A blue cube will be drawn from a bag containing only five blue and five black cubes.
  - d. A red cube will be drawn from a bag of 100 red cubes.
  - e. A red cube will be drawn from a bag of 10 red and 90 blue cubes.



3. A shape will be randomly drawn from the box shown below. Decide where each event would be located on the probability scale. Then, place the letter for each event on the appropriate place on the probability scale.



Event:

- A. A circle is drawn.
- B. A square is drawn.
- C. A star is drawn.
- D. A shape that is not a square is drawn.

## **Probability Scale**

0		1/2		1
Impossible	Unlikely	Equally Likely to	Likely	Certain
		Occur or Not Occur		

4. Color the squares below so that it would be equally likely to choose a blue or yellow square.





5. Color the squares below so that it would be likely but not certain to choose a blue square from the bag.



6. Color the squares below so that it would be unlikely but not impossible to choose a blue square from the bag.



7. Color the squares below so that it would be impossible to choose a blue square from the bag.





1. Play a game using the two spinners below. Spin each spinner once, and then multiply the outcomes together. If the result is less than or equal to 8, you win the game. Play the game 15 times, and record your results in the table below. Then, answer the questions that follow.



Turn	First Spin Results	Second Spin Results	Product
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			

- a. What is your estimate for the probability of getting a product of 8 or less?
- b. What is your estimate for the probability of getting a product of more than 8?
- c. What is your estimate for the probability of getting a product of exactly 8?
- d. What is the most likely product for this game?
- e. If you play this game another 15 times, will you get the exact same results? Explain.



2. A seventh-grade student surveyed students at her school. She asked them to name their favorite pets. Below is a bar graph showing the results of the survey.



Use the results from the survey to answer the following questions.

- a. How many students answered the survey question?
- b. How many students said that a snake was their favorite pet?

Now, suppose a student is randomly selected and asked what his favorite pet is.

- c. What is your estimate for the probability of that student saying that a dog is his favorite pet?
- d. What is your estimate for the probability of that student saying that a gerbil is his favorite pet?
- e. What is your estimate for the probability of that student saying that a frog is his favorite pet?



3. A seventh-grade student surveyed 25 students at her school. She asked them how many hours a week they spend playing a sport or game outdoors. The results are listed in the table below.

Number of Hours	Tally	Frequency
0		3
1		4
2	++++	5
3	++++	7
4		3
5		0
6		2
7		0
8		1

a. Draw a dot plot of the results.



Suppose a student will be randomly selected.

- b. What is your estimate for the probability of that student answering 3 hours?
- c. What is your estimate for the probability of that student answering 8 hours?
- d. What is your estimate for the probability of that student answering 6 or more hours?
- e. What is your estimate for the probability of that student answering 3 or fewer hours?
- f. If another 25 students were surveyed, do you think they would give the exact same results? Explain your answer.
- g. If there are 200 students at the school, what is your estimate for the number of students who would say they play a sport or game outdoors 3 hours per week? Explain your answer.



4. A student played a game using one of the spinners below. The table shows the results of 15 spins. Which spinner did the student use? Give a reason for your answer.

Spin	Results
1	1
2	1
3	2
4	3
5	1
6	2
7	3
8	2
9	2
10	1
11	2
12	2
13	1
14	3
15	1





Lesson 2:

- 1. For each of the following chance experiments, list the sample space (all the possible outcomes).
  - a. Rolling a 4-sided die with the numbers 1–4 on the faces of the die
  - b. Selecting a letter from the word *mathematics*
  - c. Selecting a marble from a bag containing 50 black marbles and 45 orange marbles
  - d. Selecting a number from the even numbers 2–14, including 2 and 14
  - e. Spinning the spinner below:



- 2. For each of the following, decide if the two outcomes listed are equally likely to occur. Give a reason for your answer.
  - a. Rolling a 1 or a 2 when a 6-sided number cube with the numbers 1–6 on the faces of the cube is rolled
  - b. Selecting the letter *a* or *k* from the word *take*
  - c. Selecting a black or an orange marble from a bag containing 50 black and 45 orange marbles
  - d. Selecting a 4 or an 8 from the even numbers 2–14, including 2 and 14
  - e. Landing on a 1 or a 3 when spinning the spinner below





3. Color the squares below so that it would be equally likely to choose a blue or yellow square.



4. Color the squares below so that it would be more likely to choose a blue than a yellow square.



5. You are playing a game using the spinner below. The game requires that you spin the spinner twice. For example, one outcome could be yellow on the 1<sup>st</sup> spin and red on the 2<sup>nd</sup> spin. List the sample space (all the possible outcomes) for the two spins.



6. List the sample space for the chance experiment of flipping a coin twice.



- 1. In a seventh-grade class of 28 students, there are 16 girls and 12 boys. If one student is randomly chosen to win a prize, what is the probability that a girl is chosen?
- 2. An experiment consists of spinning the spinner once.
  - a. Find the probability of landing on a 2.
  - b. Find the probability of landing on a 1.
  - c. Is landing in each section of the spinner equally likely to occur? Explain.



- 3. An experiment consists of randomly picking a square section from the board shown below.
  - a. Find the probability of choosing a triangle.
  - b. Find the probability of choosing a star.
  - c. Find the probability of choosing an empty square.
  - d. Find the probability of choosing a circle.



- 4. Seventh graders are playing a game where they randomly select two integers 0–9, inclusive, to form a two-digit number. The same integer might be selected twice.
  - a. List the sample space for this chance experiment. List the probability of each outcome in the sample space.
  - b. What is the probability that the number formed is between 90 and 99, inclusive?
  - c. What is the probability that the number formed is evenly divisible by 5?
  - d. What is the probability that the number formed is a factor of 64?
- 5. A chance experiment consists of flipping a coin and rolling a number cube with the numbers 1–6 on the faces of the cube.
  - a. List the sample space of this chance experiment. List the probability of each outcome in the sample space.
  - b. What is the probability of getting a heads on the coin and the number 3 on the number cube?
  - c. What is the probability of getting a tails on the coin and an even number on the number cube?



6. A chance experiment consists of spinning the two spinners below.



- a. List the sample space and the probability of each outcome.
- b. Find the probability of the event of getting a red on the first spinner and a red on the second spinner.
- c. Find the probability of a red on at least one of the spinners.



1. The Gator Girls is a soccer team. The possible number of goals the Gator Girls will score in a game and their probabilities are shown in the table below.

Number of Goals	0	1	2	3	4
Probability	0.22	0.31	0.33	0.11	0.03

Find the probability that the Gator Girls:

- a. Score more than two goals
- b. Score at least two goals
- c. Do not score exactly 3 goals
- 2. The diagram below shows a spinner. The pointer is spun, and the player is awarded a prize according to the color on which the pointer stops.



- a. What is the probability that the pointer stops in the red region?
- b. Complete the table below showing the probabilities of the three possible results.

Color	Red	Green	Blue
Probability			

- c. Find the probability that the pointer stops on green or blue.
- d. Find the probability that the pointer does not stop on green.



3. Wayne asked every student in his class how many siblings (brothers and sisters) they had. The survey results are shown in the table below. (Wayne included himself in the results.)

Number of Siblings	0	1	2	3	4
Number of Students	4	5	14	6	3

(Note: The table tells us that 4 students had no siblings, 5 students had one sibling, 14 students had two siblings, and so on.)

- a. How many students are there in Wayne's class, including Wayne?
- b. What is the probability that a randomly selected student does not have any siblings? Write your answer as a fraction in lowest terms.
- c. The table below shows the possible number of siblings and the probabilities of each number. Complete the table by writing the probabilities as fractions in lowest terms.

Number of Siblings	0	1	2	3	4
Probability					

- d. Writing your answers as fractions in lowest terms, find the probability that the student:
  - i. Has fewer than two siblings
  - ii. Has two or fewer siblings
  - iii. Does not have exactly one sibling



- 1. Imagine that a family of three (Alice, Bill, and Chester) plays bingo at home every night. Each night, the chance that any one of the three players will win is  $\frac{1}{2}$ .
  - a. Using A for Alice wins, B for Bill wins, and C for Chester wins, develop a tree diagram that shows the nine possible outcomes for two consecutive nights of play.
  - b. Is the probability that "Bill wins both nights" the same as the probability that "Alice wins the first night and Chester wins the second night"? Explain.
- According to the Washington, D.C. Lottery's website for its Cherry Blossom Doubler instant scratch game, the chance of winning a prize on a given ticket is about 17%. Imagine that a person stops at a convenience store on the way home from work every Monday and Tuesday to buy a scratcher ticket to play the game. (Source: DC Lottery)
  - a. Develop a tree diagram showing the four possible outcomes of playing over these two days. Call stage 1 "Monday," and use the symbols W for a winning ticket and L for a non-winning ticket.
  - b. What is the chance that the player will not win on Monday but will win on Tuesday?
  - c. What is the chance that the player will win at least once during the two-day period?

## Image of Tetrahedral Die

Source: http://commons.wikimedia.org/wiki/File:4-sided\_dice\_250.jpg

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- According to the Washington, D.C. Lottery's website for its Cherry Blossom Double instant scratch game, the chance
  of winning a prize on a given ticket is about 17%. Imagine that a person stops at a convenience store on the way
  home from work every Monday, Tuesday, and Wednesday to buy a scratcher ticket and plays the game.
  (Source: DC Lottery)
  - a. Develop a tree diagram showing the eight possible outcomes of playing over these three days. Call stage one "Monday," and use the symbols W for a winning ticket and L for a non-winning ticket.
  - b. What is the probability that the player will not win on Monday but will win on Tuesday and Wednesday?
  - c. What is the probability that the player will win at least once during the 3-day period?
- 2. A survey company is interested in conducting a statewide poll prior to an upcoming election. They are only interested in talking to registered voters.

Imagine that 55% of the registered voters in the state are male and 45% are female. Also, consider that the distribution of ages may be different for each group. In this state, 30% of male registered voters are age 18–24, 37% are age 25–64, and 33% are 65 or older. 32% of female registered voters are age 18–24, 26% are age 25–64, and 42% are 65 or older.

The following tree diagram describes the distribution of registered voters. The probability of selecting a male registered voter age 18-24 is 0.165.



- a. What is the chance that the polling company will select a registered female voter age 65 or older?
- b. What is the chance that the polling company will select any registered voter age 18–24?



- 1. If you created a stack of 15 pennies taped together, do you think the probability of getting a heads on a toss of the stack would be different than for a stack of 10 pennies? Explain your answer.
- 2. If you created a stack of 20 pennies taped together, what do you think the probability of getting a heads on a toss of the stack would be? Explain your answer.
- 3. Based on your work in this lesson, complete the following table of the relative frequencies of heads for the stack you created:

Number of	Total Number of Heads So Far	Relative Frequency of Heads So Far (to the nearest hundredth)
103365		
1		
5		
10		
15		
20		
25		
30		

- 4. What is your estimate of the probability that your stack of pennies will land heads up when tossed? Explain your answer.
- 5. Is there a theoretical probability you could use to compare to the estimated probability? Explain your answer.



Jerry and Michael played a game similar to *Picking Blue*! The following results are from their research using the same two bags:

Jerry's research:

Michael's research:

	Number of Red Chips Picked	Number of Blue Chips Picked		Number of Red Chips Picked	Number of Blue Chips Picked
Bag A	2	8	Bag A	28	12
Bag B	3	7	Bag B	22	18

- 1. If all you knew about the bags were the results of Jerry's research, which bag would you select for the game? Explain your answer.
- 2. If all you knew about the bags were the results of Michael's research, which bag would you select for the game? Explain your answer.
- 3. Does Jerry's research or Michael's research give you a better indication of the makeup of the blue and red chips in each bag? Explain why you selected this research.
- 4. Assume there are 12 chips in each bag. Use either Jerry's or Michael's research to estimate the number of red and blue chips in each bag. Then, explain how you made your estimates.

Bag A	Bag B
Number of red chips:	Number of red chips:
Number of blue chips:	Number of blue chips:

5. In a different game of *Picking Blue!*, two bags each contain red, blue, green, and yellow chips. One bag contains the same number of red, blue, green, and yellow chips. In the second bag, half of the chips are blue. Describe a plan for determining which bag has more blue chips than any of the other colors.



1. A mouse is placed at the start of the maze shown below. If it reaches station B, it is given a reward. At each point where the mouse has to decide which direction to go, assume that it is equally likely to go in either direction. At each decision point 1, 2, 3, it must decide whether to go left (L) or right (R). It cannot go backward.



- a. Create a theoretical model of probabilities for the mouse to arrive at terminal points A, B, and C.
  - i. List the possible paths of a sample space for the paths the mouse can take. For example, if the mouse goes left at decision point 1 and then right at decision point 2, then the path would be denoted LR.
  - ii. Are the paths in your sample space equally likely? Explain.
  - iii. What are the theoretical probabilities that a mouse reaches terminal points A, B, and C? Explain.
- b. Based on the following set of simulated paths, estimate the probabilities that the mouse arrives at points A, B, and C.

RR	RR	RL	LL	LR	RL	LR	LL	LR	RR
LR	RL	LR	RR	RL	LR	RR	LL	RL	RL
LL	LR	LR	LL	RR	RR	RL	LL	RR	LR
RR	LR	RR	LR	LR	LL	LR	RL	RL	LL

c. How do the simulated probabilities in part (b) compare to the theoretical probabilities of part (a)?



Suppose that a dartboard is made up of the 8 × 8 grid of squares shown below. Also, suppose that when a dart is thrown, it is equally likely to land on any one of the 64 squares. A point is won if the dart lands on one of the 16 black squares. Zero points are earned if the dart lands in a white square.



- a. For one throw of a dart, what is the probability of winning a point? Note that a point is won if the dart lands on a black square.
- b. Lin wants to use a number cube to simulate the result of one dart. She suggests that 1 on the number cube could represent a win. Getting 2, 3, or 4 could represent no point scored. She says that she would ignore getting a 5 or 6. Is Lin's suggestion for a simulation appropriate? Explain why you would use it, or if not, how you would change it.
- Suppose a game consists of throwing a dart three times. A trial consists of three rolls of the number cube.
   Based on Lin's suggestion in part (b) and the following simulated rolls, estimate the probability of scoring two points in three darts.

324	332	411	322	124
224	221	241	111	223
321	332	112	433	412
443	322	424	412	433
144	322	421	414	111
242	244	222	331	224
113	223	333	414	212
431	233	314	212	241
421	222	222	112	113
212	413	341	442	324

- d. The theoretical probability model for winning 0, 1, 2, and 3 points in three throws of the dart as described in this problem is:
  - i. Winning 0 points has a probability of 0.42.
  - ii. Winning 1 point has a probability of 0.42.
  - iii. Winning 2 points has a probability of 0.14.
  - iv. Winning 3 points has a probability of 0.02.

Use the simulated rolls in part (c) to build a model of winning 0, 1, 2, and 3 points, and compare it to the theoretical model.



- 1. A model airplane has two engines. It can fly if one engine fails but is in serious trouble if both engines fail. The engines function independently of one another. On any given flight, the probability of a failure is 0.10 for each engine. Design a simulation to estimate the probability that the airplane will be in serious trouble the next time it goes up.
  - a. How would you simulate the status of an engine?
  - b. What constitutes a trial for this simulation?
  - c. What constitutes a success for this simulation?
  - d. Carry out 50 trials of your simulation, list your results, and calculate an estimate of the probability that the airplane will be in serious trouble the next time it goes up.
- 2. In an effort to increase sales, a cereal manufacturer created a really neat toy that has six parts to it. One part is put into each box of cereal. Which part is in a box is not known until the box is opened. You can play with the toy without having all six parts, but it is better to have the complete set. If you are really lucky, you might only need to buy six boxes to get a complete set. But if you are very unlucky, you might need to buy many, many boxes before obtaining all six parts.
  - a. How would you represent the outcome of purchasing a box of cereal, keeping in mind that there are six different parts? There is one part in each box.
  - b. If it was stated that a customer would have to buy at least 10 boxes of cereal to collect all six parts, what constitutes a trial in this problem?
  - c. What constitutes a success in a trial in this problem?
  - d. Carry out 15 trials, list your results, and compute an estimate of the probability that it takes the purchase of 10 or more boxes to get all six parts.
- 3. Suppose that a type A blood donor is needed for a certain surgery. Carry out a simulation to answer the following question: If 40% of donors have type A blood, what is an estimate of the probability that it will take at least four donors to find one with type A blood?
  - a. How would you simulate a blood donor having or not having type A?
  - b. What constitutes a trial for this simulation?
  - c. What constitutes a success for this simulation?
  - d. Carry out 15 trials, list your results, and compute an estimate for the probability that it takes at least four donors to find one with type A blood.



 Some M&M's<sup>®</sup> are "defective." For example, a defective M&M<sup>®</sup> may have its *M* missing, or it may be cracked, broken, or oddly shaped. Is the probability of getting a defective M&M<sup>®</sup> higher for peanut M&M's<sup>®</sup> than for plain M&M's<sup>®</sup>?

Gloriann suggests the probability of getting a defective plain M&M<sup>®</sup> is the same as the probability of getting a defective peanut M&M<sup>®</sup>. Suzanne does not think this is correct because a peanut M&M<sup>®</sup> is bigger than a plain M&M<sup>®</sup>, and therefore has a greater opportunity to be damaged.

- a. Simulate inspecting a plain M&M<sup>®</sup> by rolling two number cubes. Let a sum of 7 or 11 represent a defective plain M&M<sup>®</sup> and the other possible rolls represent a plain M&M<sup>®</sup> that is not defective. Do 50 trials, and compute an estimate of the probability that a plain M&M<sup>®</sup> is defective. Record the 50 outcomes you observed. Explain your process.
- b. Simulate inspecting a peanut M&M<sup>®</sup> by selecting a card from a well-shuffled deck of cards. Let a one-eyed face card and clubs represent a defective peanut M&M<sup>®</sup> and the other cards represent a peanut M&M<sup>®</sup> that is not defective. Be sure to replace the chosen card after each trial and to shuffle the deck well before choosing the next card. Note that the one-eyed face cards are the king of diamonds, jack of hearts, and jack of spades. Do 20 trials, and compute an estimate of the probability that a peanut M&M<sup>®</sup> is defective. Record the list of 20 cards that you observed. Explain your process.
- c. For this problem, suppose that the two simulations provide accurate estimates of the probability of a defective M&M<sup>®</sup> for plain and peanut M&M's<sup>®</sup>. Compare your two probability estimates, and decide whether Gloriann's belief is reasonable that the defective probability is the same for both types of M&M's<sup>®</sup>. Explain your reasoning.



One at a time, mice are placed at the start of the maze shown below. There are four terminal stations at A, B, C, and D. At each point where a mouse has to decide in which direction to go, assume that it is equally likely for it to choose any of the possible directions. A mouse cannot go backward.

In the following simulated trials, L stands for left, R for right, and S for straight. Estimate the probability that a mouse finds station C where the food is. No food is at A, B, or D. The following data were collected on 50 simulated paths that the mice took.

RRRLRLLSLSRLRRRLRLLRLRLRLRLRLSRLLRRRLSRLRRRLLRLLLSRLRLRLRRRRRRLRLLLSRRRLRLRRRRRRLLLLRRRRLSRRRRRRLLRRLS



- a. What paths constitute a success, and what paths constitute a failure?
- b. Use the data to estimate the probability that a mouse finds food. Show your calculation.
- c. Paige suggests that it is equally likely that a mouse gets to any of the four terminal stations. What does your simulation suggest about whether her equally likely model is believable? If it is not believable, what do your data suggest is a more believable model?
- d. Does your simulation support the following theoretical probability model? Explain.
  - i. The probability a mouse finds terminal point A is 0.167.
  - ii. The probability a mouse finds terminal point B is 0.167.
  - iii. The probability a mouse finds terminal point C is 0.417.
  - iv. The probability a mouse finds terminal point D is 0.250.



## The History of the U.S. Census

The word *census* is Latin in origin and means to tax. The first U.S. census took place over 200 years ago, but the United States is certainly not the first country to implement a census. Based on archaeological records, it appears that the ancient Egyptians conducted a census as early as 3000 B.C.E.

The U.S. census is mandated by the U.S. Constitution in Article I, Section 2, which states, in part, "Representatives and direct Taxes shall be apportioned among the several States ... according to their respective Numbers .... The Number of Representatives shall not exceed one for every thirty thousand, but each State shall have at Least one Representative ...." The Constitution then specifies how to calculate the number of people in each state and how often the census should take place.

The U.S. census has been conducted every ten years since 1790, but as time has passed, our census has evolved. Not only have the types of questions changed but also the manner in which the data are collected and tabulated. Originally, the census had only a few questions, the purpose of which was to discern the number of people in each household and their ages. Presumably, these data were used to determine the number of men in each state who were available to go to war. Federal marshals were charged with the task of conducting this first census. After collecting data from their respective jurisdictions, the marshals sent the data to President Washington.

As time has passed, more questions have been added to the U.S. census. Today, the census includes questions designed to collect data in various fields such as manufacturing, commerce, and transportation, to name a few. Data that were once manually tabulated are now processed by computers. Home visits by census officials were once the norm, but now the census is conducted primarily through the U.S. Postal Service. Each household in the United States receives in the mail a copy of the census questionnaire to be completed by its head of household who then mails it back to the Census Bureau. Home visits are paid only to those individuals who do not return the questionnaire by the specified deadline.

The census is an important part of our Constitution. Today, the census not only tells us the population of each state, thereby determining the number of representatives that each state will have in the House of Representatives, but it also provides the U.S. government with very useful data that paint a picture of the current state of our population and how it has changed over the decades.

"U.S. Census History," *essortment*, accessed November 4, 2014, <u>http://www.essortment.com/census-history-20901.html</u>.



- 1. The lunch program at Blake Middle School is being revised to align with the new nutritional standards that reduce calories and increase servings of fruits and vegetables. The administration decided to do a census of all students at Blake Middle School by giving a survey to all students about the school lunches.
  - a. Name some questions that you would include in the survey. Explain why you think those questions would be important to ask.
  - b. Read through the paragraph below that describes some of the survey results. Then, identify the population characteristics and the sample statistics.

About  $\frac{3}{4}$  of the students surveyed eat the school lunch regularly. The median number of days per month that students at Blake Middle School ate a school lunch was 18 days. 36% of students responded that their favorite fruit is bananas. The survey results for Tanya's seventh-grade homeroom showed that the median number of days per month that her classmates ate lunch at school was 22, and only 20% liked bananas. The fiesta salad was approved by 78% of the group of students who tried it, but when it was put on the lunch menu, only 40% of the students liked it. Of the seventh graders as a whole, 73% liked spicy jicama strips, but only 2 out of 5 of all the middle school students liked them.

- 2. For each of the following questions, (1) describe how you would collect data to answer the question, and (2) describe whether it would result in a sample statistic or a population characteristic.
  - a. Where should the eighth-grade class go for its class trip?
  - b. What is the average number of pets per family for families that live in your town?
  - c. If people tried a new diet, what percentage would have an improvement in cholesterol reading?
  - d. What is the average grade point of students who got accepted to a particular state university?
  - e. What is a typical number of home runs hit in a particular season for major league baseball players?
- 3. Identify a question that would lead to collecting data from the given set as a population and a question where the data could be a sample from a larger population.
  - a. All students in your school
  - b. Your state
- 4. Suppose that researchers sampled attendees of a certain movie and found that the mean age was 17 years old. Based on this observation, which of the following would be most likely?
  - a. The mean age of all of the people who went to see the movie was 17 years old.
  - b. About a fourth of the people who went to see the movie were older than 51.
  - c. The mean age of all people who went to see the movie would probably be in an interval around 17 years of age, that is, between 15 and 19.
  - d. The median age of those who attended the movie was 17 years old as well.



- 5. The headlines proclaimed: "Education Impacts Work-Life Earnings Five Times More Than Other Demographic Factors, Census Bureau Reports." According to a U.S. Census Bureau study, education levels had more effect on earnings over a 40-year span in the workforce than any other demographic factor. www.census.gov/newsroom/releases/archives/education/cb11-153.html
  - a. The article stated that the estimated impact on annual earnings between a professional degree and an eighth-grade education was roughly five times the impact of gender, which was \$13,000. What would the difference in annual earnings be with a professional degree and with an eighth-grade education?
  - b. Explain whether you think the data are from a population or a sample, and identify either the population characteristic or the sample statistic.



## \1\ Casey at the Bat

The Outlook wasn't brilliant for the Mudville nine that day: The score stood four to two, **\2\** with but one inning more to play. And then when Cooney died at first, and Barrows did the same, A **\3\** sickly silence fell upon the patrons of the game.

A straggling few got up to go in deep despair. The **\4\** rest Clung to that hope which springs eternal in the human breast; They thought, if only Casey could get but **\5\** a whack at that—We'd put up even money, now, with Casey at the bat.

But Flynn preceded Casey, as **\6**\ did also Jimmy Blake, And the former was a lulu and the latter was a cake; So upon that stricken **\7**\ multitude grim melancholy sat, For there seemed but little chance of Casey's getting to the bat.

But Flynn let drive **\8\** a single, to the wonderment of all, And Blake, the much despised, tore the cover off the ball; And when **\9\** the dust had lifted, and the men saw what had occurred, There was Jimmy safe at second and Flynn a **\10\** hugging third.

Then from five thousand throats and more there rose a lusty yell; It rumbled through the valley, it **\11** rattled in the dell; It knocked upon the mountain and recoiled upon the flat, For Casey, mighty Casey, was advancing **\12** to the bat.

There was ease in Casey's manner as he stepped into his place; There was pride in Casey's **\13**\ bearing and a smile on Casey's face. And when, responding to the cheers, he lightly doffed his hat, No stranger **\14**\ in the crowd could doubt 'twas Casey at the bat.

Ten thousand eyes were on him as he rubbed his **\15\** hands with dirt; Five thousand tongues applauded when he wiped them on his shirt. Then while the writhing pitcher ground **\16\** the ball into his hip, Defiance gleamed in Casey's eye, a sneer curled Casey's lip.

And now the leather covered **\17\** sphere came hurtling through the air, And Casey stood a-watching it in haughty grandeur there. Close by the sturdy batsman **\18\** the ball unheeded sped—"That ain't my style," said Casey. "Strike one," the umpire said.

From the benches, black with **\19**\ people, there went up a muffled roar, Like the beating of the storm waves on a stern and distant shore. **\20**\ "Kill him! Kill the umpire!" shouted someone on the stand; And it's likely they'd a-killed him had not Casey raised **\21**\ his hand.



With a smile of Christian charity great Casey's visage shone; He stilled the rising tumult; he bade the **\22\** game go on; He signaled to the pitcher, and once more the spheroid flew; But Casey still ignored it, and **\23\**the umpire said, "Strike two."

"Fraud!" cried the maddened thousands, and echo answered fraud; But one scornful look from Casey **\24\** and the audience was awed. They saw his face grow stern and cold, they saw his muscles strain, And they **\25\** knew that Casey wouldn't let that ball go by again.

The sneer is gone from Casey's lip, his teeth are **\26\** clenched in hate; He pounds with cruel violence his bat upon the plate. And now the pitcher holds the ball, **\27\** and now he lets it go, And now the air is shattered by the force of Casey's blow.

Oh, somewhere **\28\** in this favored land the sun is shining bright; The band is playing somewhere, and somewhere hearts are light, And **\29\** somewhere men are laughing, and somewhere children shout; But there is no joy in Mudville—mighty Casey has struck out.

#### by Ernest Lawrence Thayer

- 1. Would any of the following provide a random sample of letters used in the text of the book *Harry Potter and the Sorcerer's Stone* by J.K. Rowling? Explain your reasoning.
  - a. Use the first letter of every word of a randomly chosen paragraph.
  - b. Number all of the letters in the words in a paragraph of the book, cut out the numbers, and put them in a bag. Then, choose a random set of numbers from the bag to identify which letters you will use.
  - c. Have a family member or friend write down a list of his favorite words, and count the number of times each of the letters occurs.
- 2. Indicate whether the following are random samples from the given population, and explain why or why not.
  - a. Population: All students in school; the sample includes every fifth student in the hall outside of class.
  - b. Population: Students in your class; the sample consists of students who have the letter s in their last names.
  - c. Population: Students in your class; the sample is selected by putting their names in a hat and drawing the sample from the hat.
  - d. Population: People in your neighborhood; the sample includes those outside in the neighborhood at 6:00 p.m.
  - e. Population: Everyone in a room; the sample is selected by having everyone toss a coin, and those that result in heads are the sample.
- 3. Consider the two sample distributions of the number of letters in randomly selected words shown below:



- a. Describe each distribution using statistical terms as much as possible.
- b. Do you think the two samples came from the same poem? Why or why not?
- 4. What questions about samples and populations might you want to ask if you saw the following headlines in a newspaper?
  - a. "Peach Pop is the top flavor according to 8 out of 10 people."
  - b. "Candidate X looks like a winner! 10 out of 12 people indicate they will vote for Candidate X."
  - c. "Students overworked. Over half of 400 people surveyed think students spend too many hours on homework."
  - d. "Action/adventure was selected as the favorite movie type by an overwhelming 75% of those surveyed."



# 100 Grocery Items (2013 prices)

T-bone steaks	Porterhouse steaks	Pasta sauce	lce cream cups
\$6.99 (1 lb.)	\$7.29 (1 lb.)	\$2.19 (16 oz.)	\$7.29 (6 cups)
Hot dog buns	Baking chips	Cheese chips	Cookies
\$0.88 (6 buns)	\$2.99 (12 oz.)	\$2.09 (12 oz.)	\$1.77 (15 oz.)
Kidney beans	Box of oatmeal	Soup	Chicken breasts
\$0.77 (15 oz.)	\$1.77 (18 oz.)	\$0.77 (14 oz.)	\$7.77 (1.5 lb.)
Pancake syrup	Cranberry juice	Asparagus	Seedless cucumbers
\$2.99 (28 oz.)	\$2.77 (64 oz.)	\$3.29 (1 lb.)	\$1.29 (1 ct.)
Avocado	Sliced pineapple	Box of tea	Cream cheese
\$1.30 (1 ct.)	\$2.99	\$4.29 (16 tea bags)	\$2.77 (16 oz.)
Italian roll	Turkey breast	Meatballs	Chili
\$1.39 (1 roll)	\$4.99 (1 lb.)	\$5.79 (26 oz.)	\$1.35 (15 oz.)
Peanut butter	Green beans	Apples	Mushrooms
\$1.63 (12 oz.)	\$0.99 (1 lb.)	\$1.99 (1 lb.)	\$0.69 (8 oz.)
Brown sugar	Confectioners' sugar	Zucchini	Yellow onions
\$1.29 (32 oz.)	\$1.39 (32 oz.)	\$0.79 (1 lb.)	\$0.99 (1 lb.)
Green peppers	Mozzarella cheese	Frozen chicken	Olive oil
\$0.99 (1 ct.)	\$2.69 (8 oz.)	\$6.49 (48 oz.)	\$2.99 (17 oz.)
Dark chocolate	Cocoa mix	Margarine	Mac and cheese
\$2.99 (9 oz.)	\$3.33 (1 package)	\$1.48 (16 oz.)	\$0.66 (6-oz. box)
Birthday cake	Crab legs	Sushi rolls	Prime rib
\$9.49 (7 in.)	\$19.99 (1 lb.)	\$12.99 (20 ct.)	\$19.99 (4 lb.)
Cooked shrimp	lce cream	Pork chops	Bananas
\$12.99 (32 oz.)	\$4.49 (1 qt.)	\$1.79 (1 lb.)	\$0.44 (1 lb.)



Chocolate milk	Beef franks	Sliced bacon	Fish fillets		
\$2.99 (1 gal.)	\$3.35 (1 lb.)	\$5.49 (1 lb.)	\$6.29 (1 lb.)		
Pears	Tangerines	Orange juice	Cherry pie		
\$1.29 (1 lb.)	\$3.99 (3 lb.)	\$2.98 (59 oz.)	\$4.44 (8 in.)		
Grapes	Peaches	Melon	Tomatoes		
\$1.28 (1 lb.)	\$1.28 (1 lb.)	\$1.69 (1 melon)	\$1.49 (1 lb.)		
Shredded cheese	Soda	Roast beef	Coffee		
\$1.88 (12 oz.)	\$0.88 (1 can)	\$6.49 (1 lb.)	\$6.49 (1 lb.)		
Feta cheese	Pickles	Loaf of rye bread	Crackers		
\$4.99 (1 lb.)	\$1.69 (12-oz. jar)	\$2.19	\$2.69 (7.9 oz.)		
Purified water	BBQ sauce	Ketchup	Chili sauce		
\$3.47 (35 pk.)	\$2.19 (24 oz.)	\$2.29 (34 oz.)	\$1.77 (12 oz.)		
Sugar	Flour	Breakfast cereal	Cane sugar		
\$1.77 (5 lb.)	\$2.11 (4 lb.)	\$2.79 (9 oz.)	\$2.39 (4 lb.)		
Cheese sticks	Cheese spread	Coffee creamer	Candy bars		
\$1.25 (10 oz.)	\$2.49 (45 oz.)	\$2.99 (12 oz.)	\$7.77 (40 oz.)		
Pudding mix	Fruit drink	Biscuit mix	Sausages		
\$0.98 (6 oz.)	\$1.11 (24 oz.)	\$0.89 (4 oz.)	\$2.38 (13 oz.)		
Ground beef	Apple juice	Ice cream sandwich	Cottage cheese		
\$4.49 (1 lb.)	\$1.48 (64 oz.)	\$1.98 (12 ct.)	\$1.98 (24 oz.)		
Frozen vegetables	English muffins	String cheese	Baby greens		
\$0.88 (10 oz.)	\$1.68 (6 ct.)	\$6.09 (24 oz.)	\$2.98 (10 oz.)		
Caramel apples	Pumpkin mix	Chicken salad	Whole wheat bread		
\$3.11 (1 ct.)	\$3.50 (1 lb.)	\$0.98 (2 oz.)	\$1.55 (1 loaf)		
Tuna	Nutrition bar	Potato chips	2% milk		
\$0.98 (2.5 oz.)	\$2.19 (1 bar)	\$2.39 (12 oz.)	\$3.13 (1 gal.)		



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1. Look at the distribution of years since the pennies were minted from Example 1. Which of the following box plots seem like they might not have come from a random sample from that distribution? Explain your thinking.



Box Plots of Three Random Samples of Penny Ages

2. Given the following sample of scores on a physical fitness test, from which of the following populations might the sample have been chosen? Explain your reasoning.



#### Dot Plots of Four Populations and One Sample



3. Consider the distribution below:



- a. What would you expect the distribution of a random sample of size 10 from this population to look like?
- b. Random samples of different sizes that were selected from the population in part (a) are displayed below. How did your answer to part (a) compare to these samples of size 10?



## **Dot Plots of Five Samples of Different Sizes**

- c. Why is it reasonable to think that these samples could have come from the above population?
- d. What do you observe about the sample distributions as the sample size increases?
- 4. Based on your random sample of prices from Exercise 6, answer the following questions:
  - a. It looks like a lot of the prices end in 9. Do your sample results support that claim? Why or why not?
  - b. What is the typical price of the items in your sample? Explain how you found the price and why you chose that method.



- 5. The sample distributions of prices for three different random samples of 25 items from a grocery store are shown below.
  - a. How do the distributions compare?

**Dot Plots of Three Samples** 



b. Thomas says that if he counts the items in his cart at that grocery store and multiplies by \$2.00, he will have a pretty good estimate of how much he will have to pay. What do you think of his strategy?



- 1. The suggestions below for how to choose a random sample of students at your school were made and vetoed. Explain why you think each was vetoed.
  - a. Use every fifth person you see in the hallway before class starts.
  - b. Use all of the students taking math the same time that your class meets.
  - c. Have students who come to school early do the activity before school starts.
  - d. Have everyone in the class find two friends to be in the sample.
- 2. A teacher decided to collect homework from a random sample of her students rather than grading every paper every day.
  - a. Describe how she might choose a random sample of five students from her class of 35 students.
  - b. Suppose every day for 75 days throughout an entire semester she chooses a random sample of five students. Do you think some students will never get selected? Why or why not?
- 3. Think back to earlier lessons in which you chose a random sample. Describe how you could have used a random number generator to select a random sample in each case.
  - a. A random sample of the words in the poem "Casey at the Bat"
  - b. A random sample of the grocery prices on a weekly flyer
- 4. Sofia decided to use a different plan for selecting a random sample of books from the population of 150 top-selling children's books from Example 2. She generated ten random numbers between 1 and 100,000 to stand for the possible number of pages in any of the books. Then, she found the books that had the number of pages specified in the sample. What would you say to Sofia?
- 5. Find an example from a newspaper, a magazine, or another source that used a sample. Describe the population, the sample, the sample statistic, how you think the sample might have been chosen, and whether or not you think the sample was random.



## Population

	0	1	2	3	4	5	6	7	8	9
00	45	58	49	78	59	36	52	39	70	51
01	50	45	45	66	71	55	65	33	60	51
02	53	83	40	51	83	57	75	38	43	77
03	49	49	81	57	42	36	22	66	68	52
04	60	67	43	60	55	63	56	44	50	58
05	64	41	67	73	55	69	63	46	50	65
06	54	58	53	55	51	74	53	55	64	16
07	28	48	62	24	82	51	64	45	41	47
08	70	50	38	16	39	83	62	50	37	58
09	79	62	45	48	42	51	67	68	56	78
10	61	56	71	55	57	77	48	65	61	62
11	65	40	56	47	44	51	38	68	64	40
12	53	22	73	62	82	78	84	50	43	43
13	81	42	72	49	55	65	41	92	50	60
14	56	44	40	70	52	47	30	9	58	53
15	84	64	64	34	37	69	57	75	62	67
16	45	58	49	78	59	36	52	39	70	51
17	50	45	45	66	71	55	65	33	60	51
18	53	83	40	51	83	57	75	38	43	77
19	49	49	81	57	42	36	22	66	68	52
20	60	67	43	60	55	63	56	44	50	58
21	64	41	67	73	55	69	63	46	50	65
22	54	58	53	55	51	74	53	55	64	16
23	28	48	62	24	82	51	64	45	41	47
24	70	50	38	16	39	83	62	50	37	58
25	79	62	45	48	42	51	67	68	56	78
26	61	56	71	55	57	77	48	65	61	62
27	65	40	56	47	44	51	38	68	64	40
28	53	22	73	62	82	78	84	50	43	43
29	81	42	72	49	55	65	41	92	50	60
30	56	44	40	70	52	47	30	9	58	53
31	84	64	64	34	37	69	57	75	62	67
32	45	58	49	78	59	36	52	39	70	51
33	50	45	45	66	71	55	65	33	60	51
34	53	83	40	51	83	57	75	38	43	77
35	49	49	81	57	42	36	22	66	68	52
36	60	67	43	60	55	63	56	44	50	58
37	64	41	67	73	55	69	63	46	50	65
38	54	58	53	55	51	74	53	55	64	16
39	28	48	62	24	82	51	64	45	41	47



Lesson 17: Sampling Variability
## Population (continued)

	0	1	2	3	4	5	6	7	8	9
40	53	70	59	62	33	31	74	44	46	68
41	37	51	84	47	46	33	53	54	70	74
42	35	45	48	45	56	60	66	60	65	57
43	42	81	67	64	60	79	46	48	67	56
44	41	21	41	58	48	38	50	53	73	38
45	35	28	43	43	55	39	75	45	68	36
46	64	31	31	40	84	79	47	63	48	46
47	34	36	54	61	33	16	50	60	52	55
48	53	52	48	47	77	37	66	51	61	64
49	40	44	45	22	36	64	50	49	64	39
50	45	69	67	33	55	61	62	38	51	43
51	55	39	46	56	53	50	44	42	40	60
52	11	36	56	69	72	73	71	48	58	52
53	81	47	36	54	81	59	50	42	80	69
54	40	43	30	54	61	13	73	65	52	40
55	71	78	71	61	54	79	63	47	49	73
56	53	70	59	62	33	31	74	44	46	68
57	37	51	84	47	46	33	53	54	70	74
58	35	45	48	45	56	60	66	60	65	57
59	42	81	67	64	60	79	46	48	67	56
60	41	21	41	58	48	38	50	53	73	38
61	35	28	43	43	55	39	75	45	68	36
62	64	31	31	40	84	79	47	63	48	46
63	34	36	54	61	33	16	50	60	52	55
64	53	52	48	47	77	37	66	51	61	64
65	40	44	45	22	36	64	50	49	64	39
66	45	69	67	33	55	61	62	38	51	43
67	55	39	46	56	53	50	44	42	40	60
68	11	36	56	69	72	73	71	48	58	52
69	81	47	36	54	81	59	50	42	80	69
70	40	43	30	54	61	13	73	65	52	40
71	71	78	71	61	54	79	63	47	49	73
72	53	70	59	62	33	31	74	44	46	68
73	37	51	84	47	46	33	53	54	70	74
74	35	45	48	45	56	60	66	60	65	57
75	42	81	67	64	60	79	46	48	67	56
76	41	21	41	58	48	38	50	53	73	38
77	35	28	43	43	55	39	75	45	68	36
78	64	31	31	40	84	79	47	63	48	46
79	34	36	54	61	33	16	50	60	52	55



## Table of Random Digits

Row																				
1	6	6	7	2	8	0	0	8	4	0	0	4	6	0	3	2	2	4	6	8
2	8	0	3	1	1	1	1	2	7	0	1	9	1	2	7	1	3	3	5	3
3	5	3	5	7	3	6	3	1	7	2	5	5	1	4	7	1	6	5	6	5
4	9	1	1	9	2	8	3	0	3	6	7	7	4	7	5	9	8	1	8	3
5	9	0	2	9	9	7	4	6	3	6	6	3	7	4	2	7	0	0	1	9
6	8	1	4	6	4	6	8	2	8	9	5	5	2	9	6	2	5	3	0	3
7	4	1	1	9	7	0	7	2	9	0	9	7	0	4	6	2	3	1	0	9
8	9	9	2	7	1	3	2	9	0	3	9	0	7	5	6	7	1	7	8	7
9	3	4	2	2	9	1	9	0	7	8	1	6	2	5	3	9	0	9	1	0
10	2	7	3	9	5	9	9	3	2	9	3	9	1	9	0	5	5	1	4	2
11	0	2	5	4	0	8	1	7	0	7	1	3	0	4	3	0	6	4	4	4
12	8	6	0	5	4	8	8	2	7	7	0	1	0	1	7	1	3	5	3	4
13	4	2	6	4	5	2	4	2	6	1	7	5	6	6	4	0	8	4	1	2
14	4	4	9	8	7	3	4	3	8	2	9	1	5	3	5	9	8	9	2	9
15	6	4	8	0	0	0	4	2	3	8	1	8	4	0	9	5	0	9	0	4
16	3	2	3	8	4	8	8	6	2	9	1	0	1	9	9	3	0	7	3	5
17	6	6	7	2	8	0	0	8	4	0	0	4	6	0	3	2	2	4	6	8
18	8	0	3	1	1	1	1	2	7	0	1	9	1	2	7	1	3	3	5	3
19	5	3	5	7	3	6	3	1	7	2	5	5	1	4	7	1	6	5	6	5
20	9	1	1	9	2	8	3	0	3	6	7	7	4	7	5	9	8	1	8	3
21	9	0	2	9	9	7	4	6	3	6	6	3	7	4	2	7	0	0	1	9
22	8	1	4	6	4	6	8	2	8	9	5	5	2	9	6	2	5	3	0	3
23	4	1	1	9	7	0	7	2	9	0	9	7	0	4	6	2	3	1	0	9
24	9	9	2	7	1	3	2	9	0	3	9	0	7	5	6	7	1	7	8	7
25	3	4	2	2	9	1	9	0	7	8	1	6	2	5	3	9	0	9	1	0
26	2	7	3	9	5	9	9	3	2	9	3	9	1	9	0	5	5	1	4	2
27	0	2	5	4	0	8	1	7	0	7	1	3	0	4	3	0	6	4	4	4
28	8	6	0	5	4	8	8	2	7	7	0	1	0	1	7	1	3	5	3	4
29	4	2	6	4	5	2	4	2	6	1	7	5	6	6	4	0	8	4	1	2
30	4	4	9	8	7	3	4	3	8	2	9	1	5	3	5	9	8	9	2	9
31	6	4	8	0	0	0	4	2	3	8	1	8	4	0	9	5	0	9	0	4
32	3	2	3	8	4	8	8	6	2	9	1	0	1	9	9	3	0	7	3	5
33	6	6	7	2	8	0	0	8	4	0	0	4	6	0	3	2	2	4	6	8
34	8	0	3	1	1	1	1	2	7	0	1	9	1	2	7	1	3	3	5	3
35	5	3	5	7	3	6	3	1	7	2	5	5	1	4	7	1	6	5	6	5
36	9	1	1	9	2	8	3	0	3	6	7	7	4	7	5	9	8	1	8	3
37	9	0	2	9	9		4	6	3	6	6	3	7	4	2	7	0	0	1	9
38	8	1	4	6	4	6	8	2	8	9	5	5	2	9	6	2	5	3	0	3
39	4	1	1	9	1	0	7	2	9	0	9	7	0	4	6	2	3	1	0	9
40	9	9	2	7	1	3	2	9	0	3	9	0	7	5	6	7	1	7	8	7



 Yousef intends to buy a car. He wishes to estimate the mean fuel efficiency (in miles per gallon) of all cars available at this time. Yousef selects a random sample of 10 cars and looks up their fuel efficiencies on the Internet. The results are shown below.

22 25 29 23 31 29 28 22 23 27

- a. Yousef will estimate the mean fuel efficiency of all cars by calculating the mean for his sample. Calculate the sample mean, and record your answer. (Be sure to show your work.)
- b. In practice, you only take one sample to estimate a population characteristic. However, if Yousef were to take another random sample of 10 cars from the same population, would he likely get the same value for the sample mean?
- c. What if Yousef were to take *many* random samples of 10 cars? Would all of the sample means be the same?
- d. Using this example, explain what sampling variability means.
- 2. Think about the mean number of siblings (brothers and sisters) for all students at your school.
  - a. What do you think is the approximate value of the mean number of siblings for the population of all students at your school?
  - b. How could you find a better estimate of this population mean?
  - c. Suppose that you have now selected a random sample of students from your school. You have asked all of the students in your sample how many siblings they have. How will you calculate the sample mean?
  - d. If you had taken a different sample, would the sample mean have taken the same value?
  - e. There are many different samples of students that you could have selected. These samples produce many different possible sample means. What is the phrase used for this concept?
  - f. Does the phrase you gave in part (e) apply only to sample means?



	0	1	2	3	4	5	6	7	8	9
0	6.18	4.67	4.01	4.06	3.28	4.47	4.86	4.91	3.96	6.18
1	4.98	5.42	5.65	2.97	2.92	7.09	2.78	4.20	5.02	4.98
2	3.12	1.89	4.19	5.12	4.38	5.34	4.22	4.27	5.25	3.12
3	3.90	4.47	4.07	4.80	6.28	5.79	6.07	7.64	6.33	3.90
4	5.55	4.99	3.77	3.63	5.21	3.85	7.43	4.72	6.53	5.55
5	4.55	5.38	5.83	4.10	4.42	5.63	5.57	5.32	5.32	4.55
6	4.56	7.67	6.39	4.05	4.51	5.16	5.29	6.34	3.68	4.56
7	5.86	4.75	4.94	3.92	4.84	4.95	4.50	4.56	7.05	5.86
8	5.00	5.47	5.00	5.70	5.71	6.19	4.41	4.29	4.34	5.00
9	5.12	5.58	6.16	6.39	5.93	3.72	5.92	4.82	6.19	5.12

1. The owner of a new coffee shop is keeping track of how much each customer spends (in dollars). One hundred of these amounts are shown in the table below. These amounts will form the *population* for this question.

a. Place the table of random digits in front of you. Select a starting point without looking at the page. Then, taking two digits at a time, select a random sample of size 10 from the population above. Write the 10 values in the space below. (For example, suppose you start at the third digit of row four of the random digit table. Taking two digits gives you 19. In the population above, go to the row labeled 1, and move across to the column labeled 9. This observation is 4.98, and that will be the first observation in your sample. Then, continue in the random digit table from the point you reached.)

Calculate the mean for your sample, showing your work. Round your answer to the nearest thousandth.

- b. Using the same approach as in part (a), select a random sample of size 20 from the population. Calculate the mean for your sample of size 20. Round your answer to the nearest thousandth.
- c. Which of your sample means is likely to be the better estimate of the population mean? Explain your answer in terms of sampling variability.



 Two dot plots are shown below. One of the dot plots shows the values of some sample means from random samples of size 10 from the population given in Problem 1. The other dot plot shows the values of some sample means from random samples of size 20 from the population given in Problem 1.



Which dot plot is for sample means from samples of size 10, and which dot plot is for sample means from samples of size 20? Explain your reasoning.

The sample means from samples of size 10 are shown in Dot Plot \_\_\_\_\_.

The sample means from samples of size 20 are shown in Dot Plot \_\_\_\_\_.

3. You are going to use a random sample to estimate the mean travel time for getting to school for all the students in your grade. You will select a random sample of students from your grade. Explain why you would like the sampling variability of the sample mean to be *small*.



1. A class of seventh graders wanted to find the proportion of M&M's<sup>®</sup> that are red. Each seventh grader took a random sample of 20 M&M's<sup>®</sup> from a very large container of M&M's<sup>®</sup>. The following is the proportion of red M&M's each student found.

0.15	0	0.1	0.1	0.05	0.1	0.2	0.05	0.1
0.1	0.15	0.2	0	0.1	0.15	0.15	0.1	0.2
0.3	0.1	0.1	0.2	0.1	0.15	0.1	0.05	0.3

- a. Construct a dot plot of the sample proportions.
- b. Describe the shape of the distribution.
- c. Describe the variability of the distribution.
- d. Suppose the seventh-grade students had taken random samples of size 50. Describe how the sampling distribution would change from the one you constructed in part (a).
- 2. A group of seventh graders wanted to estimate the proportion of middle school students who suffer from allergies. The members of one group of seventh graders each took a random sample of 10 middle school students, and the members of another group of seventh graders each took a random sample of 40 middle school students. Below are two sampling distributions of the sample proportions of middle school students who said that they suffer from allergies. Which dot plot is based on random samples of size 40? How can you tell?

#### Dot Plot A:

#### **Dot Plot of Sample Proportion**



Dot Plot B:

#### **Dot Plot of Sample Proportion**





Lesson 19: Understanding Variability When Estimating a Population Proportion

- 3. The nurse in your school district would like to study the proportion of middle school students who usually get at least eight hours of sleep on school nights. Suppose each student in your class plans on taking a random sample of 20 middle school students from your district, and each calculates a sample proportion of students who said that they usually get at least eight hours of sleep on school nights.
  - a. Do you expect everyone in your class to get the same value for their sample proportions? Explain.
  - b. Suppose each student in class increased the sample size from 20 to 40. Describe how you could reduce the sampling variability.

Understanding Variability When Estimating a Population Proportion



## **Table of Random Digits**

Row																				
1	6	6	7	2	8	0	0	8	4	0	0	4	6	0	3	2	2	4	6	8
2	8	0	3	1	1	1	1	2	7	0	1	9	1	2	7	1	3	3	5	3
3	5	3	5	7	3	6	3	1	7	2	5	5	1	4	7	1	6	5	6	5
4	9	1	1	9	2	8	3	0	3	6	7	7	4	7	5	9	8	1	8	3
5	9	0	2	9	9	7	4	6	3	6	6	3	7	4	2	7	0	0	1	9
6	8	1	4	6	4	6	8	2	8	9	5	5	2	9	6	2	5	3	0	3
7	4	1	1	9	7	0	7	2	9	0	9	7	0	4	6	2	3	1	0	9
8	9	9	2	7	1	3	2	9	0	3	9	0	7	5	6	7	1	7	8	7
9	3	4	2	2	9	1	9	0	7	8	1	6	2	5	3	9	0	9	1	0
10	2	7	3	9	5	9	9	3	2	9	3	9	1	9	0	5	5	1	4	2
11	0	2	5	4	0	8	1	7	0	7	1	3	0	4	3	0	6	4	4	4
12	8	6	0	5	4	8	8	2	7	7	0	1	0	1	7	1	3	5	3	4
13	4	2	6	4	5	2	4	2	6	1	7	5	6	6	4	0	8	4	1	2
14	4	4	9	8	7	3	4	3	8	2	9	1	5	3	5	9	8	9	2	9
15	6	4	8	0	0	0	4	2	3	8	1	8	4	0	9	5	0	9	0	4
16	3	2	3	8	4	8	8	6	2	9	1	0	1	9	9	3	0	7	3	5
17	6	6	7	2	8	0	0	8	4	0	0	4	6	0	3	2	2	4	6	8
18	8	0	3	1	1	1	1	2	7	0	1	9	1	2	7	1	3	3	5	3
19	5	3	5	7	3	6	3	1	7	2	5	5	1	4	7	1	6	5	6	5
20	9	1	1	9	2	8	3	0	3	6	7	7	4	7	5	9	8	1	8	3
21	9	0	2	9	9	7	4	6	3	6	6	3	7	4	2	7	0	0	1	9
22	8	1	4	6	4	6	8	2	8	9	5	5	2	9	6	2	5	3	0	3
23	4	1	1	9	7	0	7	2	9	0	9	7	0	4	6	2	3	1	0	9
24	9	9	2	7	1	3	2	9	0	3	9	0	7	5	6	7	1	7	8	7
25	3	4	2	2	9	1	9	0	7	8	1	6	2	5	3	9	0	9	1	0
26	2	7	3	9	5	9	9	3	2	9	3	9	1	9	0	5	5	1	4	2
27	0	2	5	4	0	8	1	7	0	7	1	3	0	4	3	0	6	4	4	4
28	8	6	0	5	4	8	8	2	7	7	0	1	0	1	7	1	3	5	3	4
29	4	2	6	4	5	2	4	2	6	1	7	5	6	6	4	0	8	4	1	2
30	4	4	9	8	7	3	4	3	8	2	9	1	5	3	5	9	8	9	2	9
31	6	4	8	0	0	0	4	2	3	8	1	8	4	0	9	5	0	9	0	4
32	3	2	3	8	4	8	8	6	2	9	1	0	1	9	9	3	0	7	3	5
33	6	6	7	2	8	0	0	8	4	0	0	4	6	0	3	2	2	4	6	8
34	8	0	3	1	1	1	1	2	7	0	1	9	1	2	7	1	3	3	5	3
35	5	3	5	7	3	6	3	1	7	2	5	5	1	4	7	1	6	5	6	5
36	9	1	1	9	2	8	3	0	3	6	7	7	4	7	5	9	8	1	8	3
37	9	0	2	9	9	7	4	6	3	6	6	3	7	4	2	7	0	0	1	9
38	8	1	4	6	4	6	8	2	8	9	5	5	2	9	6	2	5	3	0	3
39	4	1	1	9	7	0	7	2	9	0	9	7	0	4	6	2	3	1	0	9
40	9	9	2	7	1	3	2	9	0	3	9	0	7	5	6	7	1	7	8	7

ID	Travel to School	Favorite Season	Allergies	Favorite School Subject	Favorite Music	Superpower
1	Car	Spring	Yes	English	Рор	Freeze time
2	Car	Summer	Yes	Music	Рор	Telepathy
3	Car	Summer	No	Science	Рор	Fly
4	Walk	Fall	No	Computers and technology	Рор	Invisibility
5	Car	Summer	No	Art	Country	Telepathy
6	Car	Summer	No	Physical education	Rap/Hip-hop	Freeze time
7	Car	Spring	No	Physical education	Рор	Telepathy
8	Car	Winter	No	Art	Other	Fly
9	Car	Summer	No	Physical education	Рор	Fly
10	Car	Spring	No	Mathematics and statistics	Рор	Telepathy
11	Car	Summer	Yes	History	Rap/Hip-hop	Invisibility
12	Car	Spring	No	Art	Rap/Hip-hop	Freeze time
13	Bus	Winter	No	Computers and technology	Rap/Hip-hop	Fly
14	Car	Winter	Yes	Social studies	Rap/Hip-hop	Fly
15	Car	Summer	No	Art	Рор	Freeze time
16	Car	Fall	No	Mathematics and statistics	Рор	Fly
17	Bus	Winter	No	Science	Rap/Hip-hop	Freeze time
18	Car	Spring	Yes	Art	Рор	Telepathy
19	Car	Fall	Yes	Science	Рор	Telepathy
20	Car	Summer	Yes	Physical education	Rap/Hip-hop	Invisibility
21	Car	Spring	Yes	Science	Рор	Invisibility
22	Car	Winter	Yes	Mathematics and statistics	Country	Invisibility
23	Car	Summer	Yes	Art	Рор	Invisibility
24	Bus	Winter	Yes	Other	Рор	Telepathy
25	Bus	Summer	Yes	Science	Other	Fly
26	Car	Summer	No	Science	Рор	Fly
27	Car	Summer	Yes	Music	Рор	Telepathy
28	Car	Summer	No	Physical education	Country	Super strength
29	Car	Fall	Yes	Mathematics and statistics	Country	Telepathy
30	Car	Summer	Yes	Physical education	Rap/Hip-hop	Telepathy
31	Boat	Winter	No	Computers and technology	Gospel	Invisibility
32	Car	Spring	No	Physical education	Рор	Fly
33	Car	Spring	No	Physical education	Рор	Fly
34	Car	Summer	No	Mathematics and statistics	Classical	Fly
35	Car	Fall	Yes	Science	Jazz	Telepathy
36	Car	Spring	No	Science	Rap/Hip-hop	Telepathy
37	Car	Summer	No	Music	Country	Telepathy
38	Bus	Winter	No	Mathematics and statistics	Рор	Fly
39	Car	Spring	No	Art	Classical	Freeze time
40	Car	Winter	Yes	Art	Рор	Fly
41	Walk	Summer	Yes	Physical education	Rap/Hip-hop	Fly
42	Bus	Winter	Yes	Physical education	Gospel	Invisibility



43	Bus	Summer	No	Art	Other	Invisibility
44	Car	Summer	Yes	Computers and technology	Other	Freeze time
45	Car	Fall	Yes	Science	Рор	Fly
46	Car	Summer	Yes	Music	Rap/Hip-hop	Fly
47	Car	Spring	No	Science	Rap/Hip-hop	Invisibility
48	Bus	Spring	No	Music	Рор	Telepathy
49	Car	Summer	Yes	Social studies	Techno/Electronic	Telepathy
50	Car	Summer	Yes	Physical education	Рор	Telepathy
51	Car	Spring	Yes	Other	Other	Telepathy
52	Car	Summer	No	Art	Рор	Fly
53	Car	Summer	Yes	Other	Рор	Telepathy
54	Car	Summer	Yes	Physical education	Rap/Hip-hop	Invisibility
55	Bus	Summer	Yes	Physical education	Other	Super strength
56	Car	Summer	No	Science	Rap/Hip-hop	Invisibility
57	Car	Winter	No	Languages	Rap/Hip-hop	Super strength
58	Car	Fall	Yes	English	Рор	Fly
59	Car	Winter	No	Science	Рор	Telepathy
60	Car	Summer	No	Art	Рор	Invisibility
61	Car	Summer	Yes	Other	Рор	Freeze time
62	Bus	Spring	No	Science	Рор	Fly
63	Car	Winter	Yes	Mathematics and statistics	Other	Freeze time
64	Car	Summer	No	Social studies	Classical	Fly
65	Car	Winter	Yes	Science	Рор	Telepathy
66	Car	Winter	No	Science	Rock and roll	Fly
67	Car	Summer	No	Mathematics and statistics	Rap/Hip-hop	Super strength
68	Car	Fall	No	Music	Rock and roll	Super strength
69	Car	Spring	No	Other	Other	Invisibility
70	Car	Summer	Yes	Mathematics and statistics	Rap/Hip-hop	Telepathy
71	Car	Winter	No	Art	Other	Fly
72	Car	Spring	Yes	Mathematics and statistics	Рор	Telepathy
73	Car	Winter	Yes	Computers and technology	Techno/Electronic	Telepathy
74	Walk	Winter	No	Physical education	Techno/Electronic	Fly
75	Walk	Summer	No	History	Rock and roll	Fly
	Skateboard/					
76	Scooter/	Winter	Yes	Computers and technology	Techno/Electronic	Freeze time
	Rollerblade					
77	Car	Spring	Yes	Science	Other	Telepathy
78	Car	Summer	No	Music	Rap/Hip-hop	Invisibility
79	Car	Summer	No	Social studies	Рор	Invisibility
80	Car	Summer	No	Other	Rap/Hip-hop	Telepathy
81	Walk	Spring	Yes	History	Rap/Hip-hop	Invisibility
82	Car	Summer	No	Art	Рор	Invisibility



83	Walk	Spring	No	Languages	Jazz	Super strength
84	Car	Fall	No	History	Jazz	Invisibility
85	Car	Summer	No	Physical education	Rap/Hip-hop	Freeze time
86	Car	Spring	No	Mathematics and statistics	Рор	Freeze time
87	Bus	Spring	Yes	Art	Рор	Telepathy
88	Car	Winter	No	Mathematics and statistics	Other	Invisibility
89	Car	Summer	Yes	Physical education	Country	Telepathy
90	Bus	Summer	No	Computers and technology	Other	Fly
91	Car	Winter	No	History	Рор	Telepathy
92	Walk	Winter	No	Science	Classical	Telepathy
93	Bicycle	Summer	No	Physical education	Рор	Invisibility
94	Car	Summer	No	English	Рор	Telepathy
95	Car	Summer	Yes	Physical education	Рор	Fly
96	Car	Winter	No	Science	Other	Freeze time
97	Car	Winter	No	Other	Rap/Hip-hop	Super strength
98	Car	Summer	Yes	Physical education	Rap/Hip-hop	Freeze time
99	Car	Spring	No	Music	Classical	Telepathy
100	Car	Spring	Yes	Science	Gospel	Telepathy
101	Car	Summer	Yes	History	Рор	Super strength
102	Car	Winter	Yes	English	Country	Freeze time
103	Car	Spring	No	Computers and technology	Other	Telepathy
104	Car	Winter	No	History	Other	Invisibility
105	Car	Fall	No	Music	Рор	Telepathy
106	Car	Fall	No	Science	Рор	Telepathy
107	Car	Winter	No	Art	Heavy metal	Fly
108	Car	Spring	Yes	Science	Rock and roll	Fly
109	Car	Fall	Yes	Music	Other	Fly
110	Car	Summer	Yes	Social studies	Techno/Electronic	Telepathy
111	Car	Spring	No	Physical education	Рор	Fly
112	Car	Summer	No	Physical education	Рор	Fly
113	Car	Summer	Yes	Social studies	Рор	Freeze time
114	Car	Summer	Yes	Computers and technology	Gospel	Freeze time
115	Car	Winter	Yes	Other	Rap/Hip-hop	Telepathy
116	Car	Summer	Yes	Science	Country	Telepathy
117	Car	Fall		Music	Country	Fly
118	Walk	Summer	No	History	Рор	Telepathy
119	Car	Spring	Yes	Art	Рор	Freeze time
120	Car	Fall	Yes	Physical education	Rap/Hip-hop	Fly
121	Car	Spring	No	Music	Rock and roll	Telepathy
122	Car	Fall	No	Art	Рор	Invisibility
123	Car	Summer	Yes	Physical education	Rap/Hip-hop	Fly
124	Walk	Summer	No	Computers and technology	Рор	Telepathy
125	Car	Fall	No	Art	Рор	Fly



126	Bicycle	Spring	No	Science	Рор	Invisibility
127	Car	Summer	No	Social studies	Gospel	Fly
128	Bicycle	Winter	No	Social studies	Rap/Hip-hop	Fly
129	Car	Summer	Yes	Mathematics and statistics	Рор	Invisibility
130	Car	Fall	Yes	Mathematics and statistics	Country	Telepathy
131	Car	Winter	Yes	Music	Gospel	Super strength
132	Rail (Train/ Tram/Subway)	Fall	Yes	Art	Other	Fly
133	Walk	Summer	No	Social studies	Рор	Invisibility
134	Car	Summer	Yes	Music	Рор	Freeze time
135	Car	Winter	No	Mathematics and statistics	Рор	Telepathy
136	Car	Fall	Yes	Music	Рор	Telepathy
137	Car	Summer	Yes	Computers and technology	Other	Freeze time
138	Car	Summer	Yes	Physical education	Рор	Telepathy
139	Car	Summer	Yes	Social studies	Other	Telepathy
140	Car	Spring	Yes	Physical education	Other	Freeze time
141	Car	Fall	Yes	Science	Country	Telepathy
142	Car	Spring	Yes	Science	Рор	Invisibility
143	Car	Summer	No	Other	Rap/Hip-hop	Freeze time
144	Car	Summer	No	Other	Other	Fly
145	Car	Summer	No	Languages	Рор	Freeze time
146	Car	Summer	Yes	Physical education	Рор	Telepathy
147	Bus	Winter	No	History	Country	Invisibility
148	Car	Spring	No	Computers and technology	Other	Telepathy
149	Bus	Winter	Yes	Science	Рор	Invisibility
150	Car	Summer	No	Social studies	Rap/Hip-hop	Invisibility
151	Car	Summer	No	Physical education	Рор	Invisibility
152	Car	Summer	Yes	Physical education	Рор	Super strength
153	Car	Summer	No	Mathematics and statistics	Рор	Fly
154	Car	Summer	No	Art	Rap/Hip-hop	Freeze time
155	Car	Winter	Yes	Other	Classical	Freeze time
156	Car	Summer	Yes	Computers and technology	Other	Telepathy
157	Car	Spring	No	Other	Рор	Freeze time
158	Car	Winter	Yes	Music	Country	Fly
159	Car	Winter	No	History	Jazz	Invisibility
160	Car	Spring	Yes	History	Рор	Fly
161	Car	Winter	Yes	Mathematics and statistics	Other	Telepathy
162	Car	Fall	No	Science	Country	Invisibility
163	Car	Winter	No	Science	Other	Fly
164	Car	Summer	No	Science	Рор	Fly
	Skateboard/					
165	Scooter/ Rollerblade	Spring	Yes	Social studies	Other	Freeze time
166	Car	Winter	Yes	Art	Rap/Hip-hop	Fly
	-				1 17 15 517	1



167	Car	Summer	Yes	Other	Рор	Freeze time
168	Car	Summer	No	English	Рор	Telepathy
169	Car	Summer	No	Other	Рор	Invisibility
170	Car	Summer	Yes	Physical education	Techno/Electronic	Freeze time
171	Car	Summer	No	Art	Рор	Telepathy
172	Car	Summer	No	Physical education	Rap/Hip-hop	Freeze time
173	Car	Winter	Yes	Mathematics and statistics	Other	Invisibility
174	Bus	Summer	Yes	Music	Рор	Freeze time
175	Car	Winter	No	Art	Рор	Fly
176	Car	Fall	No	Science	Rap/Hip-hop	Fly
177	Car	Winter	Yes	Social studies	Рор	Telepathy
178	Car	Fall	No	Art	Other	Fly
179	Bus	Spring	No	Physical education	Country	Fly
180	Car	Winter	No	Music	Other	Telepathy
181	Bus	Summer	No	Computers and technology	Rap/Hip-hop	Freeze time
182	Car	Summer	Yes	Physical education	Rap/Hip-hop	Invisibility
183	Car	Summer	Yes	Music	Other	Telepathy
184	Car	Spring	No	Science	Rap/Hip-hop	Invisibility
185	Rail (Train/ Tram/Subway)	Summer	No	Physical education	Other	Freeze time
186	Car	Summer	Yes	Mathematics and statistics	Rap/Hip-hop	Fly
187	Bus	Winter	Yes	Mathematics and statistics	Other	Super strength
188	Car	Summer	No	Mathematics and statistics	Other	Freeze time
189	Rail (Train/ Tram/Subway)	Fall	Yes	Music	Jazz	Fly
190	Car	Summer	Yes	Science	Рор	Super strength
191	Car	Summer	Yes	Science	Techno/Electronic	Freeze time
192	Car	Spring	Yes	Physical education	Rap/Hip-hop	Freeze time
193	Car	Summer	Yes	Physical education	Rap/Hip-hop	Freeze time
194	Car	Winter	No	Physical education	Rap/Hip-hop	Telepathy
195	Car	Winter	No	Music	Jazz	Freeze time
196	Walk	Summer	Yes	History	Country	Freeze time
197	Car	Spring	No	History	Rap/Hip-hop	Freeze time
198	Car	Fall	Yes	Other	Рор	Freeze time
199	Car	Spring	Yes	Science	Other	Freeze time
200	Bicycle	Winter	Yes	Other	Rap/Hip-hop	Freeze time



1. A class of 30 seventh graders wanted to estimate the proportion of middle school students who played a musical instrument. Each seventh grader took a random sample of 25 middle school students and asked each student whether or not she played a musical instrument. The following are the sample proportions the seventh graders found in 30 samples.

0.80	0.64	0.72	0.60	0.60	0.72	0.76	0.68	0.72	0.68
0.72	0.68	0.68	0.76	0.84	0.60	0.80	0.72	0.76	0.80
0.76	0.60	0.80	0.84	0.68	0.68	0.70	0.68	0.64	0.72

- a. The first student reported a sample proportion of 0.80. What does this value mean in terms of this scenario?
- b. Construct a dot plot of the 30 sample proportions.
- c. Describe the shape of the distribution.
- d. Describe the variability of the distribution.
- e. Using the 30 class sample proportions listed above, what is your estimate for the proportion of all middle school students who played a musical instrument?
- 2. Select another variable or column from the data file that is of interest. Take a random sample of 30 students from the list, and record the response to your variable of interest of each of the 30 students.
  - a. Based on your random sample, what is your estimate for the proportion of all middle school students?
  - b. If you selected a second random sample of 30, would you get the same sample proportion for the second random sample that you got for the first random sample? Explain why or why not.



Below are three dot plots. Each dot plot represents the differences in sample means for random samples selected from two populations (Bag A and Bag B). For each distribution, the differences were found by subtracting the sample means of Bag B from the sample means of Bag A (sample mean A – sample mean B).

1. Does the graph below indicate that the population mean of Bag A is larger than the population mean of Bag B? Why or why not?



- 2. Use the graph above to estimate the difference in the population means (Mean A Mean B).
- 3. Does the graph below indicate that the population mean of Bag A is larger than the population mean of Bag B? Why or why not?



4. Does the graph below indicate that the population mean of Bag A is larger than the population mean of Bag B? Why or why not?



- 5. In the above graph, how many differences are greater than 0? How many differences are less than 0? What might this tell you?
- 6. In Problem 4, the population mean for Bag A is really larger than the population mean for Bag B. Why is it possible to still get so many negative differences in the graph?



- 1. A school is trying to decide which reading program to purchase.
  - a. How many MADs separate the mean reading comprehension score for a standard program (mean = 67.8, MAD = 4.6, n = 24) and an activity-based program (mean = 70.3, MAD = 4.5, n = 27)?
  - b. What recommendation would you make based on this result?
- 2. Does a football filled with helium go farther than one filled with air? Two identical footballs were used: one filled with helium and one filled with air to the same pressure. Matt was chosen from your team to do the kicking. You did not tell Matt which ball he was kicking. The data (in yards) follow.

Air	25	23	28	29	27	32	24	26	22	27	31	24	33	26	24	28	30
Helium	24	19	25	25	22	24	28	31	22	26	24	23	22	21	21	23	25

	Mean	MAD
Air		
Helium		

- a. Calculate the difference between the sample mean distance for the football filled with air and for the one filled with helium.
- b. On the same scale, draw dot plots of the two distributions, and discuss the variability in each distribution.
- c. Calculate the MAD for each distribution. Based on the MADs, compare the variability in each distribution. Is the variability about the same? Interpret the MADs in the context of the problem.
- d. Based on your calculations, is the difference in mean distance meaningful? Part of your reasoning should involve the number of MADs that separate the sample means. Note that if the MADs differ, use the larger one in determining how many MADs separate the two means.
- 3. Suppose that your classmates were debating about whether going to college is really worth it. Based on the following data of annual salaries (rounded to the nearest thousands of dollars) for college graduates and high school graduates with no college experience, does it appear that going to college is indeed worth the effort? The data are from people in their second year of employment.

College Grad	41	67	53	48	45	60	59	55	52	52	50	59	44	49	52
High School Grad	23	33	36	29	25	43	42	38	27	25	33	41	29	33	35

- a. Calculate the difference between the sample mean salary for college graduates and for high school graduates.
- b. On the same scale, draw dot plots of the two distributions, and discuss the variability in each distribution.
- c. Calculate the MAD for each distribution. Based on the MADs, compare the variability in each distribution. Is the variability about the same? Interpret the MADs in the context of the problem.
- d. Based on your calculations, is going to college worth the effort? Part of your reasoning should involve the number of MADs that separate the sample means.



# Copy of the Excel student data file

ID Number	Texting	ReacTime	Homework	Sleep
1	99	0.33	9.0	8.2
2	69	0.39	8.6	7.5
3	138	0.36	6.1	8.7
4	100	0.40	7.9	7.8
5	116	0.28	5.1	8.8
6	112	0.38	6.5	7.9
7	79	0.35	6.5	8.8
8	111	0.41	8.8	8.5
9	115	0.49	8.4	8.4
10	82	0.43	8.7	8.8
11	136	0.46	7.2	8.4
12	112	0.51	8.3	9.0
13	101	0.42	7.0	8.8
14	89	0.38	5.6	8.3
15	120	0.35	7.2	8.2
16	144	0.36	3.9	8.8
17	131	0.26	9.0	8.9
18	126	0.39	7.0	8.5
19	118	0.37	9.2	8.7
20	83	0.34	7.4	8.6
21	120	0.20	4.5	8.7
22	114	0.38	6.0	8.6
23	90	0.25	7.0	8.4
24	116	0.36	5.8	8.4
25	108	0.36	8.9	8.1
26	89	0.31	8.4	8.8
27	124	0.44	6.3	8.3
28	121	0.32	5.2	8.0
29	104	0.30	6.7	8.1
30	110	0.39	7.8	8.1
31	119	0.36	8.5	8.0
32	113	0.40	5.3	9.4
33	106	0.36	5.7	8.6
34	119	0.33	5.9	8.4
35	129	0.38	6.2	9.0
36	95	0.44	7.9	8.3
37	126	0.41	7.2	8.6



Lesson 23: Using Sample Data to Compare the Means of Two or More Populations

38      106      0.26      7.1      8.5        39      116      0.34      4.9      8.4        40      107      0.35      9.3      8.1        41      108      0.48      8.1      8.6        42      97      0.40      7.1      8.8        43      97      0.27      4.2      8.3        44      100      0.24      6.2      8.9        45      123      0.50      8.1      8.6        46      94      0.39      5.2      8.3        47      87      0.37      8.0      8.3        48      93      0.42      9.7      8.1        49      117      0.39      7.9      8.3        50      94      0.36      6.9      8.9        51      124      0.29      8.1      8.4        52      116      0.44      4.9      8.2        53      137      0.25      9.1      8.3        54      123      0.30					
39      116      0.34      4.9      8.4        40      107      0.35      9.3      8.1        41      108      0.48      8.1      8.6        42      97      0.40      7.1      8.8        43      97      0.27      4.2      8.3        44      100      0.24      6.2      8.9        45      123      0.50      8.1      8.6        46      94      0.39      5.2      8.3        47      87      0.37      8.0      8.3        48      93      0.42      9.7      8.1        49      117      0.39      7.9      8.3        50      94      0.36      6.9      8.9        51      124      0.29      8.1      8.4        52      116      0.44      4.9      8.2        53      137      0.25      9.1      8.3        54      123      0.30      3.8      8.7        55      122      0.21	38	106	0.26	7.1	8.5
401070.359.38.1411080.488.18.642970.407.18.843970.274.28.3441000.246.28.9451230.508.18.646940.395.28.347870.378.08.348930.429.78.1491170.397.98.350940.366.98.9511240.298.18.4521160.444.98.2531370.259.18.3541230.303.88.7551220.216.68.856920.417.68.6571010.368.58.2581110.346.58.7591260.315.68.660810.337.38.4611180.358.48.3621130.377.78.4631140.496.98.6641240.348.58.165900.486.68.766990.396.58.5671550.406.59.068770.436.78.9711070.426.9<	39	116	0.34	4.9	8.4
41      108      0.48      8.1      8.6        42      97      0.40      7.1      8.8        43      97      0.27      4.2      8.3        44      100      0.24      6.2      8.9        45      123      0.50      8.1      8.6        46      94      0.39      5.2      8.3        47      87      0.37      8.0      8.3        48      93      0.42      9.7      8.1        49      117      0.39      7.9      8.3        50      94      0.36      6.9      8.9        51      124      0.29      8.1      8.4        52      116      0.44      4.9      8.2        53      137      0.25      9.1      8.3        54      123      0.30      3.8      8.7        55      122      0.21      6.6      8.8        56      92      0.41      7.6      8.6        57      101      0.36<	40	107	0.35	9.3	8.1
42970.407.18.843970.274.28.3441000.246.28.9451230.508.18.646940.395.28.347870.378.08.348930.429.78.1491170.397.98.350940.366.98.9511240.298.18.4521160.444.98.2531370.259.18.3541230.303.88.7551220.216.68.856920.417.68.6571010.368.58.2581110.346.58.7591260.315.68.660810.337.38.4611180.358.48.3621130.377.78.4631140.496.98.6641240.348.58.165900.486.68.766990.396.58.5671550.406.59.068770.454.98.369790.237.78.6711070.426.98.6721120.396.3 </td <td>41</td> <td>108</td> <td>0.48</td> <td>8.1</td> <td>8.6</td>	41	108	0.48	8.1	8.6
43970.274.28.3441000.246.28.9451230.508.18.646940.395.28.347870.378.08.348930.429.78.1491170.397.98.350940.366.98.9511240.298.18.4521160.444.98.2531370.259.18.3541230.303.88.7551220.216.68.856920.417.68.6571010.368.58.2581110.346.58.7591260.315.68.660810.337.38.4611180.358.48.3621130.377.78.4631140.496.98.6641240.348.58.165900.486.68.766990.396.58.5671550.406.59.068770.454.98.369790.237.78.6711070.426.98.6731470.347.88.3 <trr>741260.529.2</trr>	42	97	0.40	7.1	8.8
441000.246.28.9451230.508.18.646940.395.28.347870.378.08.348930.429.78.1491170.397.98.350940.366.98.9511240.298.18.4521160.444.98.2531370.259.18.3541230.303.88.7551220.216.68.856920.417.68.6571010.368.58.2581110.346.58.7591260.315.68.660810.337.38.4611180.358.48.3621130.377.78.4631140.496.98.6641240.348.58.165900.486.68.766990.396.58.5671550.406.59.068770.454.98.369790.237.78.670910.436.78.9711070.426.98.6731470.347.88.3 <trr>741260.529.2</trr>	43	97	0.27	4.2	8.3
451230.508.18.646940.395.28.347870.378.08.348930.429.78.1491170.397.98.350940.366.98.9511240.298.18.4521160.444.98.2531370.259.18.3541230.303.88.7551220.216.68.856920.417.68.6571010.368.58.2581110.346.58.7591260.315.68.660810.337.38.4611180.358.48.3621130.377.78.4631140.496.98.6641240.348.58.165900.486.68.766990.396.58.5671550.406.59.068770.454.98.370910.436.78.9711070.426.98.6731470.347.88.3741260.529.28.9751060.276.78.0741260.529.2	44	100	0.24	6.2	8.9
46940.395.28.347870.378.08.348930.429.78.1491170.397.98.350940.366.98.9511240.298.18.4521160.444.98.2531370.259.18.3541230.303.88.7551220.216.68.856920.417.68.6571010.368.58.2581110.346.58.7591260.315.68.660810.337.38.4611180.358.48.3621130.377.78.4631140.496.98.6641240.348.58.165900.486.68.766990.396.58.5671550.406.59.068770.454.98.369790.237.78.670910.436.78.9711070.426.98.6731470.347.88.3741260.529.28.9751060.276.78.0741260.529.2<	45	123	0.50	8.1	8.6
47870.378.08.348930.429.78.1491170.397.98.350940.366.98.9511240.298.18.4521160.444.98.2531370.259.18.3541230.303.88.7551220.216.68.856920.417.68.6571010.368.58.2581110.346.58.7591260.315.68.660810.337.38.4611180.358.48.3621130.377.78.4631140.496.98.6641240.348.58.165900.486.68.766990.396.58.5671550.406.59.068770.454.98.369790.237.78.670910.436.78.9711070.426.98.6731470.347.88.3741260.529.28.9751060.276.78.0741260.529.28.9751060.276.7	46	94	0.39	5.2	8.3
48930.429.78.1491170.397.98.350940.366.98.9511240.298.18.4521160.444.98.2531370.259.18.3541230.303.88.7551220.216.68.856920.417.68.6571010.368.58.2581110.346.58.7591260.315.68.660810.337.38.4611180.358.48.3621130.377.78.4631140.496.98.6641240.348.58.165900.486.68.766990.396.58.5671550.406.59.068770.454.98.369790.237.78.670910.436.78.9711070.426.98.6731470.347.88.3741260.529.28.9751060.276.78.0741260.529.28.9751060.276.78.0741180.415.	47	87	0.37	8.0	8.3
491170.397.98.350940.366.98.9511240.298.18.4521160.444.98.2531370.259.18.3541230.303.88.7551220.216.68.856920.417.68.6571010.368.58.2581110.346.58.7591260.315.68.660810.337.38.4611180.358.48.3621130.377.78.4631140.496.98.6641240.348.58.165900.486.68.766990.396.58.5671550.406.59.068770.454.98.369790.237.78.670910.436.78.9711070.426.98.6721120.396.38.6731470.347.88.3741260.529.28.9751060.276.78.076860.216.18.4771110.297.48.0	48	93	0.42	9.7	8.1
50940.366.98.9511240.298.18.4521160.444.98.2531370.259.18.3541230.303.88.7551220.216.68.856920.417.68.6571010.368.58.2581110.346.58.7591260.315.68.660810.337.38.4611180.358.48.3621130.377.78.4631140.496.98.6641240.348.58.165900.486.68.766990.396.58.5671550.406.59.068770.454.98.369790.237.78.670910.436.78.9711070.426.98.6731470.347.88.3741260.529.28.9751060.276.78.076860.216.18.4771110.297.48.0	49	117	0.39	7.9	8.3
511240.298.18.4521160.444.98.2531370.259.18.3541230.303.88.7551220.216.68.856920.417.68.6571010.368.58.2581110.346.58.7591260.315.68.660810.337.38.4611180.358.48.3621130.377.78.4631140.496.98.6641240.348.58.165900.486.68.766990.396.58.5671550.406.59.068770.454.98.369790.237.78.670910.436.78.9711070.426.98.6731470.347.88.3741260.529.28.9751060.276.78.076860.216.18.4771110.297.48.0	50	94	0.36	6.9	8.9
521160.444.98.2531370.259.18.3541230.303.88.7551220.216.68.856920.417.68.6571010.368.58.2581110.346.58.7591260.315.68.660810.337.38.4611180.358.48.3621130.377.78.4631140.496.98.6641240.348.58.165900.486.68.766990.396.58.5671550.406.59.068770.454.98.369790.237.78.670910.436.78.9711070.426.98.6731470.347.88.3741260.529.28.9751060.276.78.076860.216.18.4771110.297.48.0781180.415.28.8	51	124	0.29	8.1	8.4
531370.259.18.3541230.303.88.7551220.216.68.856920.417.68.6571010.368.58.2581110.346.58.7591260.315.68.660810.337.38.4611180.358.48.3621130.377.78.4631140.496.98.6641240.348.58.165900.486.68.766990.396.58.5671550.406.59.068770.454.98.369790.237.78.670910.436.78.9711070.426.98.6731470.347.88.3741260.529.28.9751060.276.78.076860.216.18.4771110.297.48.0781180.415.28.8	52	116	0.44	4.9	8.2
541230.303.88.7551220.216.68.856920.417.68.6571010.368.58.2581110.346.58.7591260.315.68.660810.337.38.4611180.358.48.3621130.377.78.4631140.496.98.6641240.348.58.165900.486.68.766990.396.58.5671550.406.59.068770.454.98.369790.237.78.670910.436.78.9711070.426.98.6731470.347.88.3741260.529.28.9751060.276.78.076860.216.18.4771110.297.48.0781180.415.28.8	53	137	0.25	9.1	8.3
551220.216.68.856920.417.68.6571010.368.58.2581110.346.58.7591260.315.68.660810.337.38.4611180.358.48.3621130.377.78.4631140.496.98.6641240.348.58.165900.486.68.766990.396.58.5671550.406.59.068770.454.98.369790.237.78.670910.436.78.9711070.426.98.6731470.347.88.3741260.529.28.9751060.276.78.076860.216.18.4771110.297.48.0781180.415.28.8	54	123	0.30	3.8	8.7
56920.417.68.6571010.368.58.2581110.346.58.7591260.315.68.660810.337.38.4611180.358.48.3621130.377.78.4631140.496.98.6641240.348.58.165900.486.68.766990.396.58.5671550.406.59.068770.454.98.369790.237.78.670910.436.78.9711070.426.98.6731470.347.88.3741260.529.28.9751060.276.78.076860.216.18.4781180.415.28.8	55	122	0.21	6.6	8.8
571010.368.58.2581110.346.58.7591260.315.68.660810.337.38.4611180.358.48.3621130.377.78.4631140.496.98.6641240.348.58.165900.486.68.766990.396.58.5671550.406.59.068770.454.98.369790.237.78.670910.436.78.9711070.426.98.6721120.396.38.6731470.347.88.3741260.529.28.9751060.276.78.076860.216.18.4771110.297.48.0781180.415.28.8	56	92	0.41	7.6	8.6
581110.346.58.7591260.315.68.660810.337.38.4611180.358.48.3621130.377.78.4631140.496.98.6641240.348.58.165900.486.68.766990.396.58.5671550.406.59.068770.454.98.369790.237.78.670910.436.78.9711070.426.98.6721120.396.38.6731470.347.88.3741260.529.28.9751060.276.78.076860.216.18.4771110.297.48.0781180.415.28.8	57	101	0.36	8.5	8.2
591260.315.68.660810.337.38.4611180.358.48.3621130.377.78.4631140.496.98.6641240.348.58.165900.486.68.766990.396.58.5671550.406.59.068770.454.98.369790.237.78.670910.436.78.9711070.426.98.6731470.347.88.3741260.529.28.9751060.276.78.076860.216.18.4771110.297.48.0781180.415.28.8	58	111	0.34	6.5	8.7
60810.337.38.4611180.358.48.3621130.377.78.4631140.496.98.6641240.348.58.165900.486.68.766990.396.58.5671550.406.59.068770.454.98.369790.237.78.670910.436.78.9711070.426.98.6721120.396.38.6731470.347.88.3741260.529.28.9751060.276.78.076860.216.18.4771110.297.48.0781180.415.28.8	59	126	0.31	5.6	8.6
611180.358.48.3621130.377.78.4631140.496.98.6641240.348.58.165900.486.68.766990.396.58.5671550.406.59.068770.454.98.369790.237.78.670910.436.78.9711070.426.98.6721120.396.38.6731470.347.88.3741260.529.28.9751060.276.78.076860.216.18.4771110.297.48.0781180.415.28.8	60	81	0.33	7.3	8.4
621130.377.78.4631140.496.98.6641240.348.58.165900.486.68.766990.396.58.5671550.406.59.068770.454.98.369790.237.78.670910.436.78.9711070.426.98.6721120.396.38.6731470.347.88.3741260.529.28.9751060.276.78.076860.216.18.4771110.297.48.0781180.415.28.8	61	118	0.35	8.4	8.3
631140.496.98.6641240.348.58.165900.486.68.766990.396.58.5671550.406.59.068770.454.98.369790.237.78.670910.436.78.9711070.426.98.6721120.396.38.6731470.347.88.3741260.529.28.9751060.276.78.076860.216.18.4771110.297.48.0781180.415.28.8	62	113	0.37	7.7	8.4
641240.348.58.165900.486.68.766990.396.58.5671550.406.59.068770.454.98.369790.237.78.670910.436.78.9711070.426.98.6721120.396.38.6731470.347.88.3741260.529.28.9751060.276.78.076860.216.18.4771110.297.48.0781180.415.28.8	63	114	0.49	6.9	8.6
65900.486.68.766990.396.58.5671550.406.59.068770.454.98.369790.237.78.670910.436.78.9711070.426.98.6721120.396.38.6731470.347.88.3741260.529.28.9751060.276.78.076860.216.18.4771110.297.48.0781180.415.28.8	64	124	0.34	8.5	8.1
66990.396.58.5671550.406.59.068770.454.98.369790.237.78.670910.436.78.9711070.426.98.6721120.396.38.6731470.347.88.3741260.529.28.9751060.276.78.076860.216.18.4771110.297.48.0781180.415.28.8	65	90	0.48	6.6	8.7
671550.406.59.068770.454.98.369790.237.78.670910.436.78.9711070.426.98.6721120.396.38.6731470.347.88.3741260.529.28.9751060.276.78.076860.216.18.4771110.297.48.0781180.415.28.8	66	99	0.39	6.5	8.5
68770.454.98.369790.237.78.670910.436.78.9711070.426.98.6721120.396.38.6731470.347.88.3741260.529.28.9751060.276.78.076860.216.18.4771110.297.48.0781180.415.28.8	67	155	0.40	6.5	9.0
69790.237.78.670910.436.78.9711070.426.98.6721120.396.38.6731470.347.88.3741260.529.28.9751060.276.78.076860.216.18.4771110.297.48.0781180.415.28.8	68	77	0.45	4.9	8.3
70910.436.78.9711070.426.98.6721120.396.38.6731470.347.88.3741260.529.28.9751060.276.78.076860.216.18.4771110.297.48.0781180.415.28.8	69	79	0.23	7.7	8.6
711070.426.98.6721120.396.38.6731470.347.88.3741260.529.28.9751060.276.78.076860.216.18.4771110.297.48.0781180.415.28.8	70	91	0.43	6.7	8.9
721120.396.38.6731470.347.88.3741260.529.28.9751060.276.78.076860.216.18.4771110.297.48.0781180.415.28.8	71	107	0.42	6.9	8.6
731470.347.88.3741260.529.28.9751060.276.78.076860.216.18.4771110.297.48.0781180.415.28.8	72	112	0.39	6.3	8.6
741260.529.28.9751060.276.78.076860.216.18.4771110.297.48.0781180.415.28.8	73	147	0.34	7.8	8.3
751060.276.78.076860.216.18.4771110.297.48.0781180.415.28.8	74	126	0.52	9.2	8.9
76860.216.18.4771110.297.48.0781180.415.28.8	75	106	0.27	6.7	8.0
771110.297.48.0781180.415.28.8	76	86	0.21	6.1	8.4
78 118 0.41 5.2 8.8	77	111	0.29	7.4	8.0
	78	118	0.41	5.2	8.8

Lesson 23:

Using Sample Data to Compare the Means of Two or More Populations



79	105	0.28	5.9	8.3
80	108	0.40	7.0	8.6
81	131	0.32	7.5	8.5
82	86	0.34	7.4	8.4
83	87	0.27	7.7	8.6
84	116	0.34	9.6	9.1
85	102	0.42	5.6	8.7
86	105	0.41	8.2	8.2
87	96	0.33	4.9	8.4
88	90	0.20	7.5	8.6
89	109	0.39	6.0	8.5
90	103	0.36	5.7	9.1
91	98	0.26	5.5	8.4
92	103	0.41	8.2	8.0
93	110	0.39	9.2	8.4
94	101	0.39	8.8	8.6
95	101	0.40	6.1	9.0
96	98	0.22	7.0	8.6
97	105	0.43	8.0	8.5
98	110	0.33	8.0	8.8
99	104	0.45	9.6	8.5
100	119	0.29	7.2	9.1
101	120	0.48	6.7	8.4
102	109	0.36	6.8	8.5
103	105	0.29	8.6	8.6
104	110	0.29	7.1	8.9
105	116	0.41	6.7	8.8
106	114	0.25	9.0	8.8
107	111	0.35	9.9	8.6
108	137	0.37	6.1	7.9
109	104	0.37	7.3	9.0
110	95	0.33	8.3	8.0
111	117	0.29	7.5	8.8
112	100	0.30	2.9	9.3
113	105	0.24	6.1	8.5
114	102	0.41	7.7	8.0
115	109	0.32	5.8	8.2
116	108	0.36	6.6	8.3
117	111	0.43	8.2	8.5
118	115	0.20	5.7	8.9
119	89	0.34	5.1	8.1



120	94	0.29	9.2	8.5
121	105	0.26	7.3	8.6
122	143	0.42	5.8	8.1
123	129	0.29	6.8	8.8
124	118	0.31	6.2	8.7
125	129	0.38	9.1	8.4
126	96	0.25	6.7	8.5
127	95	0.27	7.7	8.3
128	43	0.22	6.3	9.2
129	64	0.23	3.7	8.5
130	38	0.41	5.2	9.1
131	46	0.35	7.1	8.3
132	56	0.32	4.7	8.9
133	41	0.45	7.2	8.8
134	55	0.27	4.1	8.5
135	57	0.40	7.5	8.6
136	66	0.41	6.9	8.0
137	62	0.39	3.8	8.5
138	49	0.36	6.4	8.5
139	33	0.21	4.8	9.0
140	38	0.34	5.6	8.5
141	75	0.19	5.0	8.2
142	68	0.25	5.4	8.4
143	60	0.47	7.4	9.1
144	63	0.33	4.4	8.2
145	48	0.28	7.1	8.2
146	49	0.36	2.7	8.5
147	72	0.44	5.6	7.6
148	54	0.51	4.3	8.7
149	65	0.38	7.7	8.5
150	72	0.40	3.4	9.1
151	51	0.16	4.9	8.4
152	64	0.16	4.4	8.5
153	43	0.34	0.1	8.8
154	57	0.38	4.4	8.2
155	72	0.51	3.2	8.4
156	37	0.46	5.3	8.6
157	50	0.33	4.1	8.2
158	41	0.46	4.5	8.9
159	63	0.40	5.0	8.7
160	51	0.33	5.4	7.9



Lesson	23:
EC330II	23.

Using Sample Data to Compare the Means of Two or More Populations

161	57	0.51	4.9	8.6
162	51	0.24	1.7	8.4
163	73	0.32	5.6	8.6
164	51	0.37	4.0	8.5
165	52	0.36	5.8	8.3
166	52	0.34	4.6	8.1
167	63	0.34	4.1	8.1
168	76	0.30	6.1	8.2
169	56	0.40	5.7	8.5
170	47	0.33	5.0	8.2
171	44	0.41	5.0	8.3
172	60	0.33	7.7	8.4
173	36	0.39	6.5	8.8
174	52	0.30	5.4	8.2
175	53	0.27	5.6	8.2
176	60	0.35	6.0	8.6
177	48	0.43	3.6	8.6
178	63	0.49	0.2	8.2
179	76	0.42	5.9	8.9
180	58	0.34	7.3	8.3
181	51	0.43	6.4	8.7
182	38	0.33	4.9	8.5
183	46	0.17	4.7	8.3
184	53	0.34	6.4	8.7
185	60	0.38	6.1	8.7
186	71	0.23	6.9	8.2
187	54	0.41	2.9	8.3
188	61	0.44	5.8	8.4
189	62	0.35	3.9	8.9
190	55	0.15	4.8	8.0
191	57	0.22	4.1	8.2
192	43	0.41	7.5	8.5
193	51	0.34	2.4	8.6
194	34	0.55	3.5	8.4
195	38	0.43	7.1	8.8
196	49	0.38	3.5	8.3
197	57	0.30	3.6	8.5
198	53	0.37	5.2	9.1
199	51	0.36	5.1	8.2
200	59	0.38	3.6	8.7
201	35	0.44	4.0	8.0



202	72	0.22	2.0	0.2
202	/ 3	0.52	3.0	0.3
203	21	0.37	2.7 A.C	0.4
204	31	0.30	4.0	0.0
205	40	0.33	9.0	8.3
206	60	0.36	6.6	8.5
207	66	0.44	4.2	8.5
208	47	0.22	4.5	8.7
209	56	0.30	4.8	8.6
210	72	0.36	2.9	8.8
211	68	0.50	6.6	8.3
212	45	0.37	7.3	8.5
213	58	0.17	4.9	9.0
214	64	0.34	3.2	8.6
215	66	0.34	2.5	8.4
216	49	0.29	5.0	8.3
217	83	0.39	2.5	8.8
218	73	0.33	3.6	8.4
219	52	0.34	3.3	8.8
220	56	0.28	8.8	8.7
221	58	0.32	5.6	8.3
222	53	0.40	5.9	8.1
223	50	0.23	4.4	8.4
224	43	0.34	3.9	8.7
225	50	0.39	4.4	8.0
226	44	0.31	4.4	8.4
227	59	0.36	6.0	9.1
228	41	0.35	3.2	8.4
229	53	0.29	6.6	8.7
230	49	0.37	5.7	8.3
231	42	0.22	8.5	8.6
232	48	0.34	3.9	8.2
233	60	0.31	6.1	8.8
234	56	0.50	2.6	8.5
235	43	0.25	5.3	8.9
236	67	0.32	6.1	8.8
237	43	0.24	8.6	8.8
238	41	0.46	5.1	8.7
239	66	0.45	4.9	8.3
240	44	0.52	4.1	8.7
241	70	0.43	6.6	8.8
242	63	0.38	7.9	8.4

### Lesson 23:

Using Sample Data to Compare the Means of Two or More Populations



243	47	0.24	3.9	8.3
244	52	0.38	5.4	8.8
245	49	0.47	4.2	8.4
246	45	0.31	8.1	8.8
247	46	0.37	1.9	8.3
248	19	0.31	6.5	8.3
249	63	0.40	6.1	8.5
250	64	0.35	5.8	8.1
251	63	0.34	6.7	8.5
252	68	0.46	6.9	8.5
253	48	0.43	8.6	8.7
254	43	0.38	4.4	8.3
255	50	0.32	4.6	8.7
256	76	0.31	4.0	8.3
257	64	0.39	5.7	8.6
258	38	0.29	6.4	8.0
259	90	0.30	7.0	8.6
260	37	0.39	4.8	8.8
261	58	0.37	6.5	8.0
262	42	0.27	4.5	8.6
263	58	0.37	6.0	8.3
264	42	0.42	7.2	8.8
265	66	0.33	12.6	8.8
266	116	0.44	8.7	7.5
267	76	0.43	9.5	6.9
268	125	0.46	9.9	6.8
269	128	0.41	9.8	6.6
270	128	0.37	11.3	7.1
271	125	0.44	6.7	7.9
272	80	0.49	10.6	7.1
273	110	0.48	9.9	7.2
274	135	0.41	9.8	7.8
275	136	0.45	8.9	7.2
276	142	0.43	10.2	8.0
277	120	0.48	10.2	7.5
278	109	0.43	10.1	7.1
279	109	0.50	10.9	7.5
280	111	0.35	11.8	7.4
281	101	0.49	8.5	7.8
282	98	0.50	11.6	7.2
283	91	0.56	10.0	7.3



284	151	0.50	7.7	6.7
285	82	0.48	14.0	7.5
286	107	0.48	9.5	7.5
287	83	0.40	12.0	7.2
288	91	0.40	9.2	7.9
289	127	0.40	9.1	7.6
290	115	0.42	11.6	6.8
291	118	0.40	9.8	7.3
292	89	0.42	10.8	7.0
293	100	0.46	11.6	7.3
294	97	0.39	8.5	7.8
295	110	0.36	11.1	7.7
296	88	0.40	9.0	6.7
297	103	0.47	11.7	6.7
298	82	0.49	10.7	7.5
299	87	0.41	8.1	7.4
300	130	0.39	9.8	7.6
301	116	0.42	9.6	7.6
302	96	0.42	11.8	7.1
303	122	0.39	7.9	7.1
304	70	0.38	11.1	7.4
305	116	0.47	8.8	7.3
306	122	0.48	9.0	7.0
307	109	0.45	10.0	7.3
308	114	0.50	10.1	7.3
309	62	0.47	11.4	7.3
310	120	0.51	9.5	6.3
311	130	0.38	10.5	7.7
312	92	0.47	11.8	7.3
313	81	0.55	7.9	7.3
314	82	0.51	10.1	7.7
315	102	0.48	10.9	6.5
316	113	0.43	10.2	7.9
317	119	0.43	8.0	6.8
318	108	0.48	8.9	7.0
319	130	0.53	8.3	7.3
320	111	0.50	9.9	6.6
321	132	0.50	11.5	6.8
322	110	0.47	10.8	7.1
323	95	0.49	10.4	7.5
324	137	0.29	9.8	7.5



Using Sample Data to Compare the Means of Two or More Populations



325	98	0.53	11.5	7.0
326	124	0.55	10.2	6.6
327	146	0.36	10.2	7.5
328	126	0.51	10.6	6.5
329	124	0.53	9.4	7.6
330	99	0.47	8.7	7.7
331	100	0.51	9.5	7.9
332	101	0.45	9.5	7.1
333	113	0.37	9.4	7.8
334	139	0.42	8.9	7.1
335	105	0.38	8.7	7.4
336	113	0.45	10.7	7.3
337	104	0.45	9.6	7.2
338	117	0.48	10.3	7.3
339	132	0.43	10.9	7.7
340	100	0.44	11.8	6.8
341	109	0.40	8.1	7.2
342	95	0.39	9.7	7.4
343	139	0.39	9.8	7.7
344	140	0.47	8.9	7.3
345	110	0.48	12.1	7.2
346	97	0.56	11.5	8.2
347	98	0.49	11.2	6.9
348	146	0.44	10.0	7.2
349	92	0.47	12.0	6.5
350	128	0.43	10.8	7.7
351	156	0.50	11.4	6.3
352	134	0.39	9.1	8.2
353	110	0.44	7.6	6.6
354	104	0.45	12.4	7.5
355	98	0.54	11.0	7.1
356	120	0.50	10.5	7.3
357	140	0.50	10.6	6.9
358	130	0.53	10.7	7.4
359	115	0.45	10.1	7.1
360	159	0.41	10.7	7.5
361	114	0.43	9.9	6.9
362	128	0.46	9.3	7.0
363	96	0.49	7.6	7.5
364	61	0.49	12.0	6.7
365	60	0.46	8.2	7.6



366	51	0.50	7.8	7.6
367	61	0.49	7.9	7.1
368	46	0.57	7.5	6.5
369	60	0.44	8.0	7.7
370	53	0.36	12.5	7.1
371	55	0.45	10.7	7.3
372	59	0.38	9.0	7.0
373	61	0.38	9.3	6.9
374	69	0.57	10.4	7.4
375	63	0.51	10.6	7.0
376	62	0.48	12.8	7.1
377	57	0.49	7.8	7.4
378	70	0.40	11.2	6.9
379	31	0.46	9.2	6.6
380	70	0.41	10.8	6.8
381	66	0.39	10.9	7.8
382	62	0.51	9.8	6.3
383	75	0.50	9.6	7.2
384	58	0.34	9.1	7.2
385	50	0.47	11.3	7.3
386	73	0.44	9.1	7.4
387	61	0.37	10.8	7.2
388	48	0.48	8.1	6.8
389	54	0.52	12.4	7.6
390	63	0.52	9.4	7.1
391	69	0.35	13.2	7.1
392	71	0.39	12.0	7.6
393	44	0.40	10.6	7.4
394	60	0.42	11.8	6.8
395	79	0.37	9.4	7.9
396	38	0.50	9.9	7.3
397	80	0.57	11.0	7.0
398	54	0.46	8.8	6.9
399	74	0.44	10.8	7.6
400	37	0.40	9.3	7.8
401	69	0.47	9.8	7.1
402	54	0.47	9.6	7.3
403	68	0.42	10.1	8.1
404	49	0.56	8.9	7.2
405	55	0.45	6.1	7.2
406	64	0.43	10.2	6.9



Using Sample Data to Compare the Means of Two or More Populations

407	83	0.41	7.3	6.6
408	36	0.46	11.5	7.3
409	44	0.43	11.0	6.7
410	65	0.44	11.1	7.0
411	77	0.39	12.1	7.7
412	33	0.44	6.9	7.1
413	45	0.47	8.2	6.9
414	70	0.53	7.3	7.2
415	77	0.44	8.9	7.2
416	53	0.46	9.2	6.8
417	60	0.49	11.0	7.4
418	86	0.44	5.8	7.8
419	49	0.55	8.4	7.2
420	50	0.45	12.3	6.5
421	64	0.41	10.7	7.2
422	57	0.45	7.0	7.1
423	56	0.40	12.1	6.5
424	41	0.45	12.7	7.2
425	50	0.50	8.3	6.8
426	63	0.45	11.6	7.4
427	44	0.43	7.7	7.1
428	51	0.42	10.3	7.5
429	51	0.50	8.7	7.1
430	54	0.43	8.4	7.2
431	45	0.44	7.0	6.9
432	65	0.46	10.5	7.5
433	60	0.45	7.4	7.1
434	52	0.42	4.1	7.1
435	50	0.49	11.2	6.9
436	61	0.52	10.7	6.7
437	42	0.43	9.2	6.8
438	42	0.50	11.4	6.8
439	66	0.47	7.9	7.2
440	65	0.43	9.2	7.1



- Based on Ken's population database, compare the amount of sleep that sixth-grade females get on average to the amount of sleep that eleventh-grade females get on average.
  Find the data for 15 sixth-grade females based on the following random ID numbers:
  65 1 67 101 106 87 85 95 120 4 64 74 102 31 128
  Find the data for 15 eleventh-grade females based on the following random ID numbers:
  348 313 297 351 294 343 275 354 311 328 274 305 288 267 301
- 2. On the same scale, draw dot plots for the two sample data sets.
- 3. Looking at the dot plots, list some observations comparing the number of hours per night that sixth graders spend sleeping and the number of hours per night that eleventh graders spend sleeping.
- 4. Calculate the mean and MAD for each of the data sets. How many MADs separate the two sample means? (Use the larger MAD to make this calculation if the sample MADs are not the same.)

	Mean (hours)	MAD (hours)
Sixth-Grade Females		
Eleventh-Grade Females		

5. Recall that if the number of MADs in the difference of two sample means is greater than or equal to 2, then it would be reasonable to think that the population means are different. Using this guideline, what can you say about the average number of hours of sleep per night for all sixth-grade females in the population compared to all eleventh-grade females in the population?



Learn, Practice, Succeed

# Eureka Math® Grade 7 Module 6

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32°

y°

28°

x°

x°

*y*°

v°

55°

x°

 Two lines meet at a point that is also the endpoint of a ray. Set up and solve the appropriate equations to determine x and y.

2. Two lines meet at a point that is also the vertex of an angle. Set up and solve the appropriate equations to determine *x* and *y*.

3. Two lines meet at a point that is also the vertex of an angle. Set up and solve an appropriate equation for *x* and *y*.

79°

19°

S

4. Set up and solve the appropriate equations for *s* and *t*.

5. Two lines meet at a point that is also the endpoint of two rays. Set up and solve the appropriate equations for *m* and *n*.



- 6. The supplement of the measurement of an angle is 16° less than three times the angle. Find the measurement of the angle and its supplement.
- 7. The measurement of the complement of an angle exceeds the measure of the angle by 25%. Find the measurement of the angle and its complement.
- 8. The ratio of the measurement of an angle to its complement is 1:2. Find the measurement of the angle and its complement.
- 9. The ratio of the measurement of an angle to its supplement is 3:5. Find the measurement of the angle and its supplement.
- 10. Let x represent the measurement of an acute angle in degrees. The ratio of the complement of x to the supplement of x is 2:5. Guess and check to determine the value of x. Explain why your answer is correct.



c°

1. Two lines meet at a point that is also the endpoint of a ray. Set up and solve an equation to find the value of *c*.

2. Two lines meet at a point that is also the endpoint of a ray. Set up and solve an equation to find the value of *a*. Explain why your answer is reasonable.

3. Two lines meet at a point that is also the endpoint of a ray. Set up and solve an equation to find the value of *w*.

4. Two lines meet at a point that is also the vertex of an angle. Set up and solve an equation to find the value of m.



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17°



Lesson 2:



5. Three lines meet at a point. Set up and solve an equation to find the value of

6. Three lines meet at a point that is also the endpoint of a ray. Set up and solve an equation to find the value of each variable in the diagram.

7. Set up and solve an equation to find the value of *x*. Find the measurement of  $\angle AOB$  and of  $\angle BOC$ .

8. Set up and solve an equation to find the value of x. Find the measurement of  $\angle AOB$  and of  $\angle BOC$ .







 $\frac{11x^{\circ}}{0}$ 

 $(2x - 15)^{\circ}$ 



С

9. Set up and solve an equation to find the value of x. Find the measurement of  $\angle AOB$  and of  $\angle BOC$ .



10. Write a verbal problem that models the following diagram. Then, solve for the two angles.





1. Two lines meet at a point. Set up and solve an equation to find the value of *x*.



2. Three lines meet at a point. Set up and solve an equation to find the value of *a*. Is your answer reasonable? Explain how you know.



3. Two lines meet at a point that is also the endpoint of two rays. Set up and solve an equation to find the values of *a* and *b*.



4. Three lines meet at a point that is also the endpoint of a ray. Set up and solve an equation to find the values of *x* and *y*.




5. Two lines meet at a point. Find the measurement of one of the vertical angles. Is your answer reasonable? Explain how you know.



 $15^{\circ}$ 

6. Three lines meet at a point that is also the endpoint of a ray. Set up and solve an equation to find the value of *y*.

- 7. Three adjacent angles are at a point. The second angle is 20° more than the first, and the third angle is 20° more than the second angle.
  - a. Find the measurements of all three angles.
  - b. Compare the expressions you used for the three angles and their combined expression. Explain how they are equal and how they reveal different information about this situation.
- 8. Four adjacent angles are on a line. The measurements of the four angles are four consecutive even numbers. Determine the measurements of all four angles.
- 9. Three adjacent angles are at a point. The ratio of the measurement of the second angle to the measurement of the first angle is 4 : 3. The ratio of the measurement of the third angle to the measurement of the second angle is 5 : 4. Determine the measurements of all three angles.
- 10. Four lines meet at a point. Solve for *x* and *y* in the following diagram.





- A STORY OF RATIOS
- 1. Four rays have a common endpoint on a line. Set up and solve an equation to find the value of *c*.



2. Lines *BC* and *EF* meet at *A*. Set up and solve an equation to find the value of *x*. Find the measurements of  $\angle EAH$  and  $\angle HAC$ .



3. Five rays share a common endpoint. Set up and solve an equation to find the value of x. Find the measurements of  $\angle DAG$  and  $\angle GAH$ .



4. Four lines meet at a point which is also the endpoint of three rays. Set up and solve an equation to find the values of *x* and *y*.





5. Two lines meet at a point that is also the vertex of a right angle. Set up and solve an equation to find the value of x. Find the measurements of  $\angle CAE$  and  $\angle BAG$ .



- 6. Five angles are at a point. The measurement of each angle is one of five consecutive, positive whole numbers.
  - a. Determine the measurements of all five angles.
  - b. Compare the expressions you used for the five angles and their combined expression. Explain how they are equivalent and how they reveal different information about this situation.
- 7. Let  $x^\circ$  be the measurement of an angle. The ratio of the measurement of the complement of the angle to the measurement of the supplement of the angle is 1:3. The measurement of the complement of the angle and the measurement of the supplement of the angle have a sum of  $180^\circ$ . Use a tape diagram to find the measurement of this angle.
- 8. Two lines meet at a point. Set up and solve an equation to find the value of *x*. Find the measurement of one of the vertical angles.



9. The difference between three times the measurement of the complement of an angle and the measurement of the supplement of that angle is 20°. What is the measurement of the angle?



Given the following triangle correspondences, use double arrows to show the correspondence between vertices, angles, and sides.

1.

Triangle Correspondence	$\triangle ABC \leftrightarrow \triangle RTS$
Correspondence of Vertices	
Correspondence of Angles	
Correspondence of Sides	

2.

Triangle Correspondence	$\triangle ABC \leftrightarrow \triangle FGE$
Correspondence of Vertices	
Correspondence of Angles	
Correspondence of Sides	

3.

Triangle Correspondence	$\triangle QRP \leftrightarrow \triangle WYX$
Correspondence of Vertices	
Correspondence of Angles	
Correspondence of Sides	



Name the angle pairs and side pairs to find a triangle correspondence that matches sides of equal length and angles of equal measurement.



- 7. Consider the following points in the coordinate plane.
  - a. How many different (non-identical) triangles can be drawn using any three of these six points as vertices?

b. How can we be sure that there are no more possible triangles?

- 8. Quadrilateral *ABCD* is identical with quadrilateral *WXYZ* with a correspondence  $A \leftrightarrow W$ ,  $B \leftrightarrow X$ ,  $C \leftrightarrow Y$ , and  $D \leftrightarrow Z$ .
  - a. In the figure below, label points *W*, *X*, *Y*, and *Z* on the second quadrilateral.



- b. Set up a correspondence between the side lengths of the two quadrilaterals that matches sides of equal length.
- c. Set up a correspondence between the angles of the two quadrilaterals that matches angles of equal measure.



Use a ruler, protractor, and compass to complete the following problems.

- 1. Draw a segment *AB* that is 5 cm in length and perpendicular to segment *CD*, which is 2 cm in length.
- 2. Draw supplementary angles so that one angle is 26°. Label each angle with its measurement.
- 3. Draw  $\triangle ABC$  so that  $\angle B$  has a measurement of 100°.
- 4. Draw a segment *AB* that is 3 cm in length. Draw a circle with center *A* and radius *AB*. Draw a second circle with diameter *AB*.
- 5. Draw an isosceles  $\triangle ABC$ . Begin by drawing  $\angle A$  with a measurement of 80°. Use the rays of  $\angle A$  as the equal legs of the triangle. Choose a length of your choice for the legs, and use your compass to mark off each leg. Label each marked point with *B* and *C*. Label all angle measurements.
- 6. Draw an isosceles  $\triangle DEF$ . Begin by drawing a horizontal segment DE that is 6 cm in length. Use your protractor to draw  $\angle D$  and  $\angle E$  so that the measurements of both angles are 30°. If the non-horizontal rays of  $\angle D$  and  $\angle E$  do not already cross, extend each ray until the two rays intersect. Label the point of intersection F. Label all side and angle measurements.
- 7. Draw a segment AB that is 7 cm in length. Draw a circle with center A and a circle with center B so that the circles are not the same size, but do intersect in two distinct locations. Label one of these intersections C. Join A to C and B to C to form  $\triangle ABC$ .
- Draw an isosceles trapezoid WXYZ with two equal base angles, ∠W and ∠X, that each measures 110°. Use your compass to create the two equal sides of the trapezoid. Leave arc marks as evidence of the use of your compass. Label all angle measurements. Explain how you constructed the trapezoid.



- 1. Draw rectangle ABCD with AB = 5 cm and BC = 7 cm.
- 2. Use a setsquare, ruler, and protractor to draw parallelogram PQRS so that the measurement of  $\angle P$  is 65°, PQ = 8 cm, the measurement of  $\angle Q$  is 115°, and the length of the altitude to  $\overline{PQ}$  is 3 cm.
- 3. Use a setsquare, ruler, and protractor to draw rhombus *ABCD* so that the measurement of  $\angle A$  is 60°, and each side of the rhombus measures 5 cm.

The following table contains partial information for parallelogram *ABCD*. Using no tools, make a sketch of the parallelogram. Then, use a ruler, protractor, and setsquare to draw an accurate picture. Finally, complete the table with the unknown lengths.

	$\angle A$	AB	Altitude to $\overline{AB}$	BC	Altitude to $\overline{BC}$
4.	45°	5 cm		4 cm	
5.	50°	3 cm		3 cm	
6.	60°	4 cm	4 cm		

- Use what you know about drawing parallel lines with a setsquare to draw trapezoid ABCD with parallel sides AB and CD. The length of AB is 3 cm, and the length of CD is 5 cm; the height between the parallel sides is 4 cm. Write a plan for the steps you will take to draw ABCD.
- 8. Use the appropriate tools to draw rectangle *FIND* with FI = 5 cm and IN = 10 cm.
- 9. Challenge: Determine the area of the largest rectangle that will fit inside an equilateral triangle with side length 5 cm.



- 1. Draw three different acute triangles XYZ, X'Y'Z', and X''Y''Z'' so that one angle in each triangle is 45°. Label all sides and angle measurements. Why are your triangles not identical?
- 2. Draw three different equilateral triangles ABC, A'B'C', and A''B''C''. A side length of  $\triangle ABC$  is 3 cm. A side length of  $\triangle A'B'C'$  is 5 cm. A side length of  $\triangle A''B''C''$  is 7 cm. Label all sides and angle measurements. Why are your triangles not identical?
- 3. Draw as many isosceles triangles that satisfy the following conditions: one angle measures 110°, and one side measures 6 cm. Label all angle and side measurements. How many triangles can be drawn under these conditions?
- 4. Draw three nonidentical triangles so that two angles measure 50° and 60° and one side measures 5 cm.
  - a. Why are the triangles not identical?
  - b. Based on the diagrams you drew for part (a) and for Problem 2, what can you generalize about the criterion of three given angles in a triangle? Does this criterion determine a unique triangle?



1. A triangle with side lengths 3 cm, 4 cm, and 5 cm exists. Use your compass and ruler to draw a triangle with the same side lengths. Leave all construction marks as evidence of your work, and label all side and angle measurements.

Under what condition is the triangle drawn? Compare the triangle you drew to two of your peers' triangles. Are the triangles identical? Did the condition determine a unique triangle? Use your construction to explain why.

- 2. Draw triangles under the conditions described below.
  - a. A triangle has side lengths 5 cm and 6 cm. Draw two nonidentical triangles that satisfy these conditions. Explain why your triangles are not identical.
  - b. A triangle has a side length of 7 cm opposite a 45° angle. Draw two nonidentical triangles that satisfy these conditions. Explain why your triangles are not identical.
- 3. Diagonal  $\overline{BD}$  is drawn in square *ABCD*. Describe what condition(s) can be used to justify that  $\triangle ABD$  is identical to  $\triangle CBD$ . What can you say about the measures of  $\angle ABD$  and  $\angle CBD$ ? Support your answers with a diagram and explanation of the correspondence(s) that exists.
- 4. Diagonals  $\overline{BD}$  and  $\overline{AC}$  are drawn in square *ABCD*. Show that  $\triangle ABC$  is identical to  $\triangle BAD$ , and then use this information to show that the diagonals are equal in length.
- 5. Diagonal  $\overline{QS}$  is drawn in rhombus *PQRS*. Describe the condition(s) that can be used to justify that  $\triangle PQS$  is identical to  $\triangle RQS$ . Can you conclude that the measures of  $\angle PQS$  and  $\angle RQS$  are the same? Support your answer with a diagram and explanation of the correspondence(s) that exists.
- 6. Diagonals  $\overline{QS}$  and  $\overline{PR}$  are drawn in rhombus *PQRS* and meet at point *T*. Describe the condition(s) that can be used to justify that  $\triangle PQT$  is identical to  $\triangle RQT$ . Can you conclude that the line segments *PR* and *QS* are perpendicular to each other? Support your answers with a diagram and explanation of the correspondence(s) that exists.



- 1. In  $\triangle FGH$ ,  $\angle F = 42^{\circ}$  and  $\angle H = 70^{\circ}$ . FH = 6 cm. Draw  $\triangle F'G'H'$  under the same condition as  $\triangle FGH$ . Leave all construction marks as evidence of your work, and label all side and angle measurements. What can you conclude about  $\triangle FGH$  and  $\triangle F'G'H'$ ? Justify your response.
- 2. In  $\triangle WXY$ ,  $\angle Y = 57^{\circ}$  and  $\angle W = 103^{\circ}$ . Side YX = 6.5 cm. Draw  $\triangle W'X'Y'$  under the same condition as  $\triangle WXY$ . Leave all construction marks as evidence of your work, and label all side and angle measurements. What can you conclude about  $\triangle WXY$  and  $\triangle W'X'Y'$ ? Justify your response.
- 3. Points *A*, *Z*, and *E* are collinear, and  $\angle B = \angle D$ . What can be concluded about  $\triangle ABZ$  and  $\triangle EDZ$ ? Justify your answer.



- 4. Draw  $\triangle ABC$  so that  $\angle A$  has a measurement of 60°,  $\angle B$  has a measurement of 60°, and  $\overline{AB}$  has a length of 8 cm. What are the lengths of the other sides?
- 5. Draw  $\triangle ABC$  so that  $\angle A$  has a measurement of 30°,  $\angle B$  has a measurement of 60°, and  $\overline{BC}$  has a length of 5 cm. What is the length of the longest side?



- 1. Decide whether each set of three given lengths determines a triangle. For any set of lengths that does determine a triangle, use a ruler and compass to draw the triangle. Label all side lengths. For sets of lengths that do not determine a triangle, write "Does not determine a triangle," and justify your response.
  - a. 3 cm, 4 cm, 5 cm
  - b. 1 cm, 4 cm, 5 cm
  - c. 1 cm, 5 cm, 5 cm
  - d. 8 cm, 3 cm, 4 cm
  - e. 8 cm, 8 cm, 4 cm
  - f. 4 cm, 4 cm, 4 cm
- 2. For each angle measurement below, provide one angle measurement that will determine a triangle and one that will not determine a triangle. Provide a brief justification for the angle measurements that will not form a triangle. Assume that the angles are being drawn to a horizontal segment AB; describe the position of the non-horizontal rays of angles  $\angle A$  and  $\angle B$ .

∠A	∠B: A Measurement That Determines a Triangle	∠B: A Measurement That Does Not Determine a Triangle	Justification for No Triangle
40°			
100°			
90°			
135°			

3. For the given side lengths, provide the minimum and maximum whole number side lengths that determine a triangle.

Given Side Lengths	Minimum Whole Number Third Side Length	Maximum Whole Number Third Side Length
5 cm, 6 cm		
3 cm, 7 cm		
4 cm, 10 cm		
1 cm, 12 cm		



In each of the triangles below, two sides and a non-included acute angle are marked. Use a compass to draw a
nonidentical triangle that has the same measurements as the marked angle and marked sides (look at Exercise 1,
part (e) of the Exploratory Challenge as a reference). Draw the new triangle on top of the old triangle. What is true
about the marked angles in each triangle that results in two non-identical triangles under this condition?



2. Sometimes two sides and a non-included angle of a triangle determine a unique triangle, even if the angle is acute. In the following two triangles, copy the marked information (i.e., two sides and a non-included acute angle), and discover which determines a unique triangle. Measure and label the marked parts.

In each triangle, how does the length of the marked side adjacent to the marked angle compare with the length of the side opposite the marked angle? Based on your drawings, specifically state when the two sides and acute non-included angle condition determines a unique triangle.





Lesson 12: Unique Triangles—Two Sides and a Non-Included Angle 3. A sub-condition of the two sides and non-included angle is provided in each row of the following table. Decide whether the information determines a unique triangle. Answer with a *yes, no,* or *maybe* (for a case that may or may not determine a unique triangle).

	Condition	Determines a Unique Triangle?
1	Two sides and a non-included $90^\circ$ angle.	
2	Two sides and an acute, non-included angle.	
3	Two sides and a non-included $140^\circ$ angle.	
4	Two sides and a non-included $20^{\circ}$ angle, where the side adjacent to the angle is shorter than the side opposite the angle.	
5	Two sides and a non-included angle.	
6	Two sides and a non-included $70^{\circ}$ angle, where the side adjacent to the angle is longer than the side opposite the angle.	

- 4. Choose one condition from the table in Problem 3 that does not determine a unique triangle, and explain why.
- 5. Choose one condition from the table in Problem 3 that does determine a unique triangle, and explain why.



In each of the following four problems, two triangles are given. State whether the triangles are *identical, not identical*, or *not necessarily identical*. If the triangles are identical, give the triangle conditions that explain why, and write a triangle correspondence that matches the sides and angles. If the triangles are not identical, explain why. If it is not possible to definitively determine whether the triangles are identical, write "the triangles are not necessarily identical," and explain your reasoning.







1.





4.







Checking for Identical Triangles

Lesson 13:

For Problems 5–8, three pieces of information are given for  $\triangle ABC$  and  $\triangle YZX$ . Draw, freehand, the two triangles (do not worry about scale), and mark the given information. If the triangles are identical, give a triangle correspondence that matches equal angles and equal sides. Explain your reasoning.

- 5. AB = YZ, BC = ZX, AC = YX
- 6.  $AB = YZ, BC = ZX, \angle C = \angle Y$
- 7.  $AB = XZ, \angle A = \angle Z, \angle C = \angle Y$
- 8.  $AB = XY, AC = YZ, \angle C = \angle Z$  (Note that both angles are obtuse.)



In the following problems, determine whether the triangles are *identical, not identical,* or *not necessarily identical*; justify your reasoning. If the relationship between the two triangles yields information that establishes a condition, describe the information. If the triangles are identical, write a triangle correspondence that matches the sides and angles.





Lesson 14: Checking for Identical Triangles



8. Are there any identical triangles in this diagram?









1. Jack is asked to cut a cake into 8 equal pieces. He first cuts it into equal fourths in the shape of rectangles, and then he cuts each rectangle along a diagonal.

Did he cut the cake into 8 equal pieces? Explain.

2. The bridge below, which crosses a river, is built out of two triangular supports. The point M lies on  $\overline{BC}$ . The beams represented by  $\overline{AM}$  and  $\overline{DM}$  are equal in length, and the beams represented by  $\overline{AB}$  and  $\overline{DC}$  are equal in length. If the supports were constructed so that  $\angle A$  and  $\angle D$  are equal in measurement, is point M the midpoint of  $\overline{BC}$ ? Explain.





A right rectangular prism is shown along with line segments that lie in a face. For each line segment, draw and give the approximate dimensions of the slice that results when the slicing plane contains the given line segment and is perpendicular to the face that contains the line segment.





a.





10 cm



В

b ≈ 11 cm

206

d.

B	10 cm	c	
6 cm			
	d ≈ 6 cm		
G		F	

e.

e = 6 cm $E$ $E$	C	8 cm	D	
	6 cm	e = 6 cm		
	F		E	

f.





g.





208

A side view of a right rectangular pyramid is given. The line segments lie in the lateral faces.



- a. For segments *n*, *s*, and *r*, sketch the resulting slice from slicing the right rectangular pyramid with a slicing plane that contains the line segment and is perpendicular to the base.
- b. For segment *m*, sketch the resulting slice from slicing the right rectangular pyramid with a slicing plane that contains the segment and is parallel to the base.

*Note:* To challenge yourself, you can try drawing the slice into the pyramid.

c. A top view of a right rectangular pyramid is given. The line segments lie in the base face. For each line segment, sketch the slice that results from slicing the right rectangular pyramid with a plane that contains the line segment and is perpendicular to the base.





Slice as a 2D shape

1. Draw a slice into the right rectangular prism at an angle in the form of the provided shape, and draw each slice as a 2D shape.

		Slice made in the prism
a.	A triangle	
b.	A quadrilateral	
c.	A pentagon	
d.	A hexagon	

EUREKA MATH 2. Draw slices at an angle in the form of each given shape into each right rectangular pyramid, and draw each slice as a 2D shape.



- 3. Why is it not possible to draw a slice in the shape of a hexagon for a right rectangular pyramid?
- 4. If the slicing plane meets every face of a right rectangular prism, then the slice is a hexagonal region. What can you say about opposite sides of the hexagon?
- 5. Draw a right rectangular prism so that rectangles ABCD and A'B'C'D' are base faces. The line segments AA', BB', CC', and DD' are edges of the lateral faces.
  - a. A slicing plane meets the prism so that vertices *A*, *B*, *C*, and *D* lie on one side of the plane, and vertices *A'*, *B'*, *C'*, and *D'* lie on the other side. Based on the slice's position, what other information can be concluded about the slice?
  - b. A slicing plane meets the prism so that vertices *A*, *B*, *C*, and *B*' are on one side of the plane, and vertices *A*', *D*', *C*', and *D* are on the other side. What other information can be concluded about the slice based on its position?



In the given three-dimensional figures, unit cubes are stacked exactly on top of each other on a tabletop. Each block is either visible or below a visible block.

1.

- a. The following three-dimensional figure is built on a tabletop. If slices parallel to the tabletop are taken of this figure, then what would each slice look like?
- b. Given the level slices in the figure, how many cubes are in the figure?



## 2.

- a. The following three-dimensional figure is built on a tabletop. If slices parallel to the tabletop are taken of this figure, then what would each slice look like?
- b. Given the level slices in the figure, how many cubes are in the figure?



3.

- a. The following three-dimensional figure is built on a tabletop. If slices parallel to the tabletop are taken of this figure, then what would each slice look like?
- b. Given the level slices in the figure, how many cubes are in the figure?
- 4. John says that we should be including the Level 0 slice when mapping slices. Naya disagrees, saying it is correct to start counting cubes from the Level 1 slice. Who is right?
- 5. Draw a three-dimensional figure made from cubes so that each successive layer farther away from the tabletop has one less cube than the layer below it. Use a minimum of three layers. Then draw the slices, and explain the connection between the two.





 A farmer has four pieces of unfenced land as shown to the right in the scale drawing where the dimensions of one side are given. The farmer trades all of the land and \$10,000 for 8 acres of similar land that is fenced. If one acre is equal to 43,560 ft<sup>2</sup>, how much per square foot for the extra land did the farmer pay rounded to the nearest cent?



- 2. An ordinance was passed that required farmers to put a fence around their property. The least expensive fences cost \$10 for each foot. Did the farmer save money by moving the farm?
- 3. A stop sign is an octagon (i.e., a polygon with eight sides) with eight equal sides and eight equal angles. The dimensions of the octagon are given. One side of the stop sign is to be painted red. If Timmy has enough paint to cover 500 ft<sup>2</sup>, can he paint 100 stop signs? Explain your answer.



4. The Smith family is renovating a few aspects of their home. The following diagram is of a new kitchen countertop. Approximately how many square feet of counter space is there?





Lesson 20:

5. In addition to the kitchen renovation, the Smiths are laying down new carpet. Everything but closets, bathrooms, and the kitchen will have new carpet. How much carpeting must be purchased for the home?



6. Jamie wants to wrap a rectangular sheet of paper completely around cans that are  $8\frac{1}{2}$  in. high and 4 in. in diameter. She can buy a roll of paper that is  $8\frac{1}{2}$  in. wide and 60 ft. long. How many cans will this much paper wrap?



1. A square with a side length of *a* units is decreased by *b* units in both length and width.



Use the diagram to express  $(a - b)^2$  in terms of the other  $a^2$ , ab, and  $b^2$  by filling in the blanks below:

- $(a-b)^{2} = a^{2} b(a-b) b(a-b) b^{2}$ =  $a^{2} - \underline{\qquad} + \underline{\qquad} - \underline{\qquad} + \underline{\qquad} - b^{2}$ =  $a^{2} - 2ab + \underline{\qquad} - b^{2}$ = \_\_\_\_\_\_
- 2. In Example 3, part (c), we generalized that  $(a + b)^2 = a^2 + 2ab + b^2$ . Use these results to evaluate the following expressions by writing 1,001 = 1,000 + 1.
  - a. Evaluate 101<sup>2</sup>.
  - b. Evaluate 1,001<sup>2</sup>.
  - c. Evaluate  $21^2$ .
- 3. Use the results of Problem 1 to evaluate  $999^2$  by writing 999 = 1,000 1.



4. The figures below show that  $8^2 - 5^2$  is equal to (8-5)(8+5).



- a. Create a drawing to show that  $a^2 b^2 = (a b)(a + b)$ .
- b. Use the result in part (a),  $a^2 b^2 = (a b)(a + b)$ , to explain why:
  - i.  $35^2 5^2 = (30)(40)$ .
  - ii.  $21^2 18^2 = (3)(39)$ .
  - iii.  $104^2 63^2 = (41)(167)$ .
- c. Use the fact that  $35^2 = (30)(40) + 5^2 = 1,225$  to create a way to mentally square any two-digit number ending in 5.
- 5. Create an area model for each product. Use the area model to write an equivalent expression that represents the area.
  - a. (x+1)(x+4) =
  - b. (x+5)(x+2) =
  - c. Based on the context of the area model, how do the expressions provided in parts (a) and (b) differ from the equivalent expression answers you found for each?
- 6. Use the distributive property to multiply the following expressions.
  - a. (2+6)(2+4)
  - b. (x + 6)(x + 4); draw a figure that models this multiplication problem.
  - c. (10+7)(10+7)
  - d. (a+7)(a+7)
  - e. (5-3)(5+3)
  - f. (x-3)(x+3)



1. A circle with center O has an area of 96 in<sup>2</sup>. Find the area of the shaded region.



Peyton's Solution	Monte's Solution
$A = \frac{1}{3} (96 \text{ in}^2) = 32 \text{ in}^2$	$A = \frac{96}{120} \operatorname{in}^2 = 0.8 \operatorname{in}^2$

Which person solved the problem correctly? Explain your reasoning.

2. The following region is bounded by the arcs of two quarter circles, each with a radius of 4 cm, and by line segments 6 cm in length. The region on the right shows a rectangle with dimensions 4 cm by 6 cm. Show that both shaded regions have equal areas.



3. A square is inscribed in a paper disc (i.e., a circular piece of paper) with a radius of 8 cm. The paper disc is red on the front and white on the back. Two edges of the circle are folded over. Write and explain a numerical expression that represents the area of the figure. Then, find the area of the figure.





4. The diameters of four half circles are sides of a square with a side length of 7 cm.



- a. Find the exact area of the shaded region.
- b. Find the approximate area using  $\pi \approx \frac{22}{7}$ .
- c. Find the area using the  $\pi$  button on your calculator and rounding to the nearest thousandth.
- 5. A square with a side length of 14 inches is shown below, along with a quarter circle (with a side of the square as its radius) and two half circles (with diameters that are sides of the square). Write and explain a numerical expression that represents the area of the figure.



6. Three circles have centers on segment AB. The diameters of the circles are in the ratio 3: 2: 1. If the area of the largest circle is 36 ft<sup>2</sup>, find the area inside the largest circle but outside the smaller two circles.





7. A square with a side length of 4 ft. is shown, along with a diagonal, a quarter circle (with a side of the square as its radius), and a half circle (with a side of the square as its diameter). Find the exact, combined area of regions I and II.





## Determine the surface area of the figures.











4.



5.





Determine the surface area of each figure.

1. In addition to the calculation of the surface area, describe how you found the surface area.








4. Determine the surface area after two square holes with a side length of 2 m are cut through the solid figure composed of two rectangular prisms.



5. The base of a right prism is shown below. Determine the surface area if the height of the prism is 10 cm. Explain how you determined the surface area.





**Opening Exercise** 





1. The pieces in Figure 1 are rearranged and put together to form Figure 2.



- a. Use the information in Figure 1 to determine the volume of the prism.
- b. Use the information in Figure 2 to determine the volume of the prism.
- c. If we were not told that the pieces of Figure 1 were rearranged to create Figure 2, would it be possible to determine whether the volumes of the prisms were equal without completing the entire calculation for each?
- 2. Two right prism containers each hold 75 gallons of water. The height of the first container is 20 inches. The of the second container is 30 inches. If the area of the base in the first container is 6 ft<sup>2</sup>, find the area of the base in the second container. Explain your reasoning.
- 3. Two containers are shaped like right rectangular prisms. Each has the same height, but the base of the larger container is 50% more in each direction. If the smaller container holds 8 gallons when full, how many gallons does the larger container hold? Explain your reasoning.
- 4. A right prism container with the base area of 4 ft<sup>2</sup> and height of 5 ft. is filled with water until it is 3 ft. deep. If a solid cube with edge length 1 ft. is dropped to the bottom of the container, how much will the water rise?
- 5. A right prism container with a base area of 10 ft<sup>2</sup> and height 9 ft. is filled with water until it is 6 ft. deep. A large boulder is dropped to the bottom of the container, and the water rises to the top, completely submerging the boulder without causing overflow. Find the volume of the boulder.
- 6. A right prism container with a base area of 8 ft<sup>2</sup> and height 6 ft. is filled with water until it is 5 ft. deep. A solid cube is dropped to the bottom of the container, and the water rises to the top. Find the length of the cube.
- 7. A rectangular swimming pool is 30 feet wide and 50 feet long. The pool is 3 feet deep at one end, and 10 feet deep at the other.
  - a. Sketch the swimming pool as a right prism.
  - b. What kind of right prism is the swimming pool?
  - c. What is the volume of the swimming pool in cubic feet?
  - d. How many gallons will the swimming pool hold if each cubic feet of water is about 7.5 gallons?
- 8. A milliliter (mL) has a volume of 1 cm<sup>3</sup>. A 250 mL measuring cup is filled to 200 mL. A small stone is placed in the measuring cup. The stone is completely submerged, and the water level rises to 250 mL.
  - a. What is the volume of the stone in cm<sup>3</sup>?
  - b. Describe a right rectangular prism that has the same volume as the stone.



1. Find the volume of the three-dimensional object composed of right rectangular prisms.



2. A smaller cube is stacked on top of a larger cube. An edge of the smaller cube measures  $\frac{1}{2}$  cm in length, while the larger cube has an edge length three times as long. What is the total volume of the object?



3. Two students are finding the volume of a prism with a rhombus base but are provided different information regarding the prism. One student receives Figure 1, while the other receives Figure 2.



- a. Find the expression that represents the volume in each case; show that the volumes are equal.
- b. How does each calculation differ in the context of how the prism is viewed?



Objects

Lesson 26:

6in

4. Find the volume of wood needed to construct the following side table composed of right rectangular prisms.

**A STORY OF RATIOS** 

 A plastic die (singular for dice) for a game has an edge length of 1.5 cm. Each face of the cube has the number of cubic cutouts as its marker is supposed to indicate (i.e., the face marked 3 has 3 cutouts). What is the volume of the die?

6. A wooden cube with an edge length of 6 inches has square holes (holes in the shape of right rectangular prisms) cut through the centers of each of the three sides as shown in the figure. Find the volume of the resulting solid if the square for the holes has an edge length of 1 inch.

- 7. A right rectangular prism has each of its dimensions (length, width, and height) increased by 50%. By what percent is its volume increased?
- 8. A solid is created by putting together right rectangular prisms. If each of the side lengths is increase by 40%, by what percent is the volume increased?

Volume of Composite Three-Dimensional







- 1. Harvey puts a container in the shape of a right rectangular prism under a spot in the roof that is leaking. Rainwater is dripping into the container at an average rate of 12 drops a minute. The container Harvey places under the leak has a length and width of 5 cm and a height of 10 cm. Assuming each raindrop is roughly 1 cm<sup>3</sup>, approximately how long does Harvey have before the container overflows?
- 2. A large square pipe has inside dimensions 3 in. × 3 in., and a small square pipe has inside dimensions 1 in. × 1 in. Water travels through each of the pipes at the same constant flow speed. If the large pipe can fill a pool in 2 hours, how long will it take the small pipe to fill the same pool?
- 3. A pool contains 12,000 ft<sup>3</sup> of water and needs to be drained. At 8:00 a.m., a pump is turned on that drains water at a flow rate of 10 ft<sup>3</sup> per minute. Two hours later, at 10:00 a.m., a second pump is activated that drains water at a flow rate of 8 ft<sup>3</sup> per minute. At what time will the pool be empty?
- 4. In the previous problem, if water starts flowing into the pool at noon at a flow rate of 3 ft<sup>3</sup> per minute, how much longer will it take to drain the pool?
- 5. A pool contains 6,000 ft<sup>3</sup> of water. Pump A can drain the pool in 15 hours, Pump B can drain it in 12 hours, and Pump C can drain it in 10 hours. How long will it take all three pumps working together to drain the pool?
- 6. A 2,000-gallon fish aquarium can be filled by water flowing at a constant rate in 10 hours. When a decorative rock is placed in the aquarium, it can be filled in 9.5 hours. Find the volume of the rock in cubic feet (1  $ft^3 = 7.5$  gal.)

